A Morphological Approach to Development of a Process for Measurement Uncertainty Estimation

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Abstract
The problem of increasing the reliability of uncertainty estimation of measurement results is considered. The purpose of this work was to justify the application of the process approach to the formation of the algorithm of uncertainty estimation on the basis of morphological analysis. It is theoretically substantiated that from the standpoint of the system approach to achieving an acceptable degree of reliability of measurement uncertainty estimates it is necessary to implement a process approach to the formation of the estimation method as an algorithm of actions. The main stages of the estimation process are defined. It is established that each stage of the estimation process can be realized by alternative methods. The morphological box method as a realization of morphological analysis is proposed as a basis for its solution. A morphological box design of the uncertainty estimation process with an open architecture is presented, based only on commonly accepted methods and approaches for realizing each step of the process. Two aspects of the application of the morphological box method are identified. On the one hand, the morphological box allows to form an algorithm of the uncertainty assessment process, maximally acceptable for the laboratory conditions, as a combination of process steps, based on the task at hand, combining different variants of realization of these steps. On the other hand, the morphological box acts as a tool for development of new methods of realization of various stages of the uncertainty assessment process. Examples of using the morphological box method to develop alternative algorithms of the uncertainty estimation process of the same measurement method and to develop new methods of realization of different stages of the estimation process are considered.

Keywords: measurement uncertainty, uncertainty estimation process, morphological box
Морфологический подход к разработке процесса оценивания неопределённости измерений

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Рассмотрена проблема повышения достоверности оценивания неопределённости результатов измерений. Целью данной работы являлось обоснование применения процессного подхода к формированию алгоритма оценивания неопределённости на базе морфологического анализа. Теоретически обосновано, что с позиций системного подхода к достижению приемлемой степени достоверности оценок неопределенности измерений следует реализовать процессный подход к формированию метода оценивания как алгоритма действий. Определены основные этапы процесса оценивания. Установлено, что каждый этап процесса оценивания может быть реализован альтернативными методами. В качестве основы для её решения предложен метод морфологического ящика как реализация морфологического анализа. Представлена конструкция морфологического ящика процесса оценивания неопределённости с открытой архитектурой, основанная только на общепринятых методах и подходах реализации каждого этапа процесса. Определены два аспекта применения метода морфологического ящика. С одной стороны, морфологический ящик позволяет формировать алгоритм процесса оценивания неопределённости, максимально приемлемый для условий лаборатории, как комбинацию этапов процесса, исходя из поставленной задачи, сочетая различные варианты реализации данных этапов. С другой стороны, морфологический ящик выступает как инструмент для разработки новых методов реализации различных этапов процесса оценивания неопределённости. Рассмотрены примеры использования метода морфологического ящика для разработки альтернативных алгоритмов процесса оценивания неопределённости одного и того же метода измерений и разработки новых методов реализации различных этапов процесса оценивания.

Ключевые слова: неопределённость измерения, процесс оценки неопределённости, морфологический ящик
Introduction

Constantly growing requirements to objectivity and reliability of information about the quality of products and services act as a stimulating factor for the development of theoretical basis of metrology. The indicator of measurement quality is, by definition, the uncertainty of the result, which determines the risk of incorrect decision making in relation to the circulation of products and services on the market. Until recently, the GUM model method was a tool for estimation of measurement uncertainty.

The emergence of standardized, i.e. generally accepted alternative methods, such as the Monte Carlo simulation method, empirical methods based on the use of the results of intralaboratory or interlaboratory studies of measurement methods, allows us to conclude that the problem of objectivity and reliability of measurement uncertainty assessment is still unsolved.

From the point of view of system analysis, the existing methods of estimation of measurement uncertainty are not really such, because its key point is the concept of process. System analysis assumes that the criterion of unambiguous solution of the problem related to the object is the consideration of the object as a process. In our case, the object of analysis is the process of measurement uncertainty estimation, which includes stages:

1. Formulation of the measurement problem:
   1.1. Definition of measurement result $Y$ as an output quantity of the measurement process.
   1.2. Identifying input variables $X_i$, affecting $Y$.
   1.3. Composing a measurement model as an expression of the type $Y = f(X_i)$.

2. Assigning a probability distribution to input values $X_i$ on the basis of available information.

3. Complexing of probabilistic characteristics of input quantities $X_i$ in accordance with the measurement model $Y = f(X_i)$ in order to obtain probabilistic characteristics of output quantity $Y$ with the subsequent estimation of the result – mathematical expectation $Y_0$, total uncertainty $u(Y)$ and expanded uncertainty $U(Y, P)$.

As a criterion of efficiency of the process of measurement uncertainty assessment, the reliability of the assessment is used. Obviously, the reliability of the assessment is determined first of all by the completeness and inexhaustibility of the whole array of influencing factors involved in all stages of the process, acting as a "skeleton" for the identification of the sources of factors. From this point of view, the existing generally recognized above mentioned methods of measurement uncertainty estimation do not "cover" the whole estimation process.

As a rule, they address only the third step of the estimation process, and often not in full. None of the above-mentioned methods solves the problem of justification of the set of input influencing quantities $X_i$, correct determination of their probabilistic characteristics, justification of the coupling function $Y = f(X_i)$.

Consequently, there is no reason to expect from the above methods of estimation of measurement uncertainty the required level of objectivity and reliability.

A correct method for estimating uncertainty in accordance with the systems approach as a "problem-solving methodology" should include the implementation of all three steps of the process, ensuring the integrity of the solution to the problem of objectivity and validity. At the same time, attention should be paid to the fact that each stage of the above assessment process, in turn, can be realized by different alternative methods. This implies that there can be many realizations of the process of measurement uncertainty estimation as algorithms for solving the problem representing the combination of different methods at each stage. This fact opens up unlimited possibilities of forming a correct and at the same time effective algorithm for estimating the uncertainty of the measurement (test) method in specific conditions of the laboratory according to the criterion of maximum feasibility.

Obviously, to solve this problem it is necessary to define a complete set of variants of realization of each of the process steps to form an algorithm for estimation of measurement uncertainty for specific conditions.
In this regard, the morphological box method is of interest as an implementation of morphological analysis [8–10]. Morphological analysis was developed for multifactorial, non-quantifiable problems, when classical cause-and-effect modeling does not give the desired result.

The morphological box method consists in dividing the object of analysis into meaningful constituent elements. For each element, a set of possible realizations contributing to the overall solution of the problem is determined, after which a complete set of solutions is formed by merging specific realizations of all constituent elements. The search for the best solution from the full set of solutions is performed in accordance with the chosen criterion of acceptability.

To solve the problem of formation of an acceptable for specific conditions algorithm of measurement uncertainty estimation with the help of morphological analysis, it is proposed to form the design of the morphological box as a table, the inputs (elements) of which correspond to the stages of measurement uncertainty estimation given above:
1 – identification of influencing factors;
2 – formation of the communication function (measurement model);
3 – definition of input values;
4 – transforming distribution laws.

**Element 1: Identifying influencing factors**

This step of the algorithm is the most critical from the point of view of the degree of confidence in the measurement results, because it is often based on the use of expert subjective opinions when identifying input influencing factors and assessing their significance. In metrological practice, a number of methods and techniques have been developed to facilitate the identification and search for influencing factors:

– recommendations of the classical theory of errors on identification of influencing factors belonging to the groups: instrumental, methodical, subjective, measurement conditions [11];
– "uncertainty ladder", which is a hierarchical scheme of the structure of measurement uncertainty sources: part factor – part series factor – individual laboratory factor – interlaboratory factor. The method provides a basis for determining and evaluating such characteristics of measurement methods as precision and bias estimates [2, 20];
– significant factors taken into account in the study of indicators of correctness and precision of

![Diagram of the morphological box method](image)
measurement methods according to STB ISO 5725-3\(^{6}\): operator, equipment, equipment calibration, environmental conditions (temperature, humidity, air pollution, etc.), time passing between measurements, etc.;


The above methods and techniques are based on the assumption that, despite the great variety of measurement tasks, it is possible to identify typical sources of variability that together make the greatest contribution to the uncertainty of the measurement result. Expert methods such as brainstorming, questionnaires, surveys, etc. are used to identify significant sources and factors.

Taking into account the subjectivity of these methods, as well as the assumptions made at this stage, there are risks associated with the identification of not all factors \(X_i\), which affect the final result of measurements.

The above methods and approaches can be, to a first approximation, included in the design of the morphological box as alternative realizations of the input "identification of influencing factors".

**Element 2. Formation of the communication function (measurement model)**

The realization of this stage is a measurement model – it is a functional dependence \(Y = f(X)\) of values of all input factors of the measurement process \(X\) with its result \(Y\). The summary measurement model as a model of approximation of the coupling function can be represented in a general form as:

\[
y = f(x_1, x_2, ..., x_m, a_1, a_2, ..., a_n),
\]

where \(Y\) is the estimated measurement result, \(x_1, x_2, ..., x_m\) – influencing elementary and complex factors (grouped data), \(f\) – analytical link function, \(a_1, a_2, ..., a_n\) – parameters of the analytical link function to be determined.

A measurement model is necessary in order to shape:

– model of the measurement result (mathematical expectations model);

– model of uncertainty of the measurement result (standard deviation model).

Various bases for classifying communication functions can be proposed. To form this element of the morphological box, it is rational to propose the following classification of measurement models:

– coupling function, known a priori and having a natural physical meaning;

– coupling function given a priori and represented as a series, e. g., Taylor series [13, 14];

– the link function, unknown a priori and presented as a regression model based on the results of the planned measurement experiment [15].

The situation when the coupling function is known a priori is typical for indirect measurements. For example, the model of indirect measurement of direct current force as a physical law has the form:

\[
I = \frac{U}{R_{sh-r}},
\]

where \(I\) is the actual value of direct current force, A; \(U\) is the actual value of measured voltage, V; \(R_{sh-r}\) is the actual value of shunt resistance, Ohm.

The situation when the coupling function is specified a priori is typical for direct measurements. The a priori unknown coupling function (1) can be decomposed into a Taylor series within the framework of realization of the model method of measurement uncertainty estimation and represented with the assumption as a linear additive model of the type:

\[
Y = X_{ind} + C + C_{12} + ... + C_i,
\]

where \(X_{ind}\) is the result of the instrument reading; \(C_i\) is the so-called corrections, e. g. influence factors of the reference, operator, working environment conditions, etc. [13, 14].

The situation when the coupling function is formed as a statistical model based on the results of a planned measurement experiment is possible in all generally recognized methods of measurement uncertainty assessment. This situation is most typical for empirical methods of measurement uncertainty estimation.

The ISO 5725 series\(^{6}\) of standards considers a statistical measurement model as a measurement model:

\[
y = m + B + e = \mu + (\delta + B) + e,
\]

where \( m = \mu + \delta \) is total mean value of measurement result (mathematical expectation); \( \mu \) is accepted reference value; \( \delta \) is measurement method bias; \( (\delta + B) = \Delta \) is laboratory bias; \( B \) is laboratory bias component; \( \delta \) is bias component occurring at each measurement according to repeatability conditions.

ISO 217484 considers a more generalized statistical model as a measurement model:

\[
y = m + B + \sum_{i=1}^{M} C_i x_i + e,
\]

where \( \sum_{i=1}^{M} C_i x_i \) are factors influencing the measurement result, not taken into account by the model presented in (3).

In terms of applying the communication function, two cases are possible:

1. If we use a modeling or Monte Carlo method for numerical estimation of measurement uncertainty by the method under study, we need the link function (1), since it is directly involved in the estimation process. To obtain it, we should organize a measurement experiment (full-factor plan of type \( 2^n \), fractional factor plan, other), perform regression analysis and obtain the link function (1) as a regression equation [12, 16].

2. If we use empirical methods to estimate the measurement uncertainty by the method under study, the coupling function (1) is not directly required. ISO 5725-3\(^6\) for this situation prescribes to organize the measurement experiment according to a hierarchical nested plan and to perform a variance analysis of the obtained results. Standard deviation in conditions of perceptibility SR is an analog of the total uncertainty of measurements by a specific method.

   The above methods can be, to a first approximation, included in the design of a morphological box table as alternative implementations of the input "input value determination" input.

**Element 3. Definition of input values**

There are two approaches to defining input quantities:

- complex, in which probabilistic characteristics, including the total standard uncertainty of the measurement result, are determined at once;
- differential, in which probabilistic characteristics, including standard uncertainties, are defined for each input quantity.

The integrated approach, as mentioned above, is characteristic of empirical methods for uncertainty assessment [1].

The differential approach assumes that the set of input quantities \( X_1, X_2, \ldots, X_N \), involved in the measurement, can be conditionally divided into two groups [13–15, 17]:

1. quantities whose values and uncertainties are determined directly in the current measurement (as a result of a single observation, repeated observations) – Type A estimation;
2. quantities, values and uncertainties of which are determined on the basis of a priori information (characteristics of used standards, reference materials, reference data, etc.) – type B estimation.

The above methods can be, to a first approximation, included in the design of a morphological box table as alternative implementations of the input "input value determination" input.

**Element 4. Transforming probabilistic characteristics of input quantities in order to estimate the measurement result and its uncertainty**

This element of the process of estimation of measurement uncertainty involves complexing the probabilistic characteristics of input quantities \( X_i \) according to the measurement model \( Y = f(X_i) \) in order to obtain the probabilistic characteristics of the output quantity \( Y \): mathematical expectation \( Y_0 \), total uncertainty \( u(Y) \), expanded uncertainty \( U(Y), P \).

Two approaches\(^5\) to transforming the characteristics of input random variables taking into account the coupling function in order to obtain the characteristics of the measurement result are generally accepted: frequency and Bayesian [2].

The frequency approach is based on obtaining point estimates (statistics), which can be used to construct a confidence interval. Among the generally accepted methods, the frequency approach implements the GUM modeling method, empirical and combined methods [2].

The GUM modeling method is implemented as an "eight-step method", and involves the element-by-element assessment of contributions to the total uncertainty of all influencing factors with their subsequent complexing according to the "law of propagation of uncertainties" [18].

Empirical methods of measurement uncertainty estimation are based on experimental data obtained from the results of intralaboratory or interlaboratory studies of the measurement method in terms of bias and precision, additional statistical processing of which allows estimating the total uncertainty of measurements by the method under study [1].
The combined method for uncertainty assessment appeared as a rational need to apply a combination of GUM modeling and empirical methods to the study of the same measurement method [19]. For example, if a measurement process involves several relatively independent operations, it is rational to estimate the uncertainty of the intermediate results of some of them by the modeling method and some of them by empirical methods, using measurement quality control data, proficiency tests, interlaboratory comparisons, or other periodic controls [9, 21].

Note that when estimating the uncertainty of the value \( Y \) by these methods, the distribution function of the input values \( X_i \) is not used explicitly. The generally accepted interpretation of the concept of probability is used.

When implementing the frequency approach by the listed methods, it should be taken into account that the methodological error of estimation \( u_c(y) \) or \( U(y) \), \( P \), associated with the failure to correctly transform the distribution laws of input quantities \( X_i \) into the distribution law of the measurement result \( Y \), has not been quantitatively evaluated by anyone, although it is, for example, brought to the forefront by critics of the GUM modeling method [18].

The so-called "Bayesian approach" is adopted as an idealized approach to transformation, for which the quantitative measure of uncertainty of the measurement result \( Y \) in the broad sense is the probability distribution of a random variable associated with the measured quantity, and in the narrow sense is the scattering parameter of the same distribution \( u_s(y) \) or \( U(y), P \).

The Bayesian approach involves transforming the distributions of the input quantities \( X_i \), which involves determining the probability distribution density of the output quantity \( Y \) based on the probability distribution densities of the input quantities \( X_i \) and the measurement model used \( Y = f(X_i) \). The latter is used to determine:

1) estimates of the mathematical expectation of the quantity \( Y \) in the form of an estimate of \( y \);
2) the coverage interval for the value \( Y \) corresponding to a given probability (coverage probability) \( P \) as the expanded uncertainty \( U(y) \);
3) estimates of the standard deviation of \( Y \) as the standard uncertainty \( u_s(y) \) associated with \( y \).

The transformation of distribution laws in the Bayesian approach can be performed numerically or analytically. Among the generally accepted methodology correct methods of finding the uncertainty in accordance there is only one – the Monte Carlo method [16]. The method numerically realizes the transformation of the distribution laws of input quantities into the distribution law of the measurement result by numerical methods, i.e. it imitates the analytical solution of the problem, since the value \( Y \) is not directly measured. Accordingly, the method requires serious software and high competence of the researcher.

The above methods can be, to a first approximation, included in the design of the morphological box table as alternative realizations of the input "transforming probabilistic characteristics of input quantities".

**Morphological box of the uncertainty assessment process**

Table 1 presents a morphological box of the uncertainty assessment process, constructed based only on commonly accepted methods and approaches for implementing each step of the uncertainty assessment process. Two positive aspects of the morphological analysis of the uncertainty assessment process can be emphasized:

– the morphological box acts as a "constructor" from which the method of uncertainty assessment can be formed step by step as a combination of process steps based on the task at hand, combining different variants of realization of these steps;

– the morphological box can be perceived as a tool for forming new methods at different stages of the uncertainty assessment process.

The use of the morphological box as a "constructor" for formation of the uncertainty assessment process, which is the most acceptable for laboratory conditions, is demonstrated in Table 2. As follows from the table, laboratory specialists consider three variants of realization of the process of uncertainty assessment of the measurement method investigated in the laboratory:

1 – within the framework of in-laboratory studies of method correctness and precision in accordance with ISO 5725-3 it is supposed to use empirical methods;

2 – within the framework of interlaboratory studies of method correctness and precision in accordance with ISO 21748 it is supposed to use empirical methods;

3 – direct application of the GUM method is assumed [18].
### Morphological box of the uncertainty assessment process

<table>
<thead>
<tr>
<th>Element (stage of the evaluation process)</th>
<th>Options</th>
</tr>
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</table>

| 2. Formation of the communication function (measurement model) | The communication function is known a priori | The coupling function is unknown a priori. It is formed as a statistical model based on the results of a measurement experiment |
|---------------------------------------------------------------|-----------------------------------------------|---------------------------------------------------------------------------------
|                                                                 | Exactly (physical model of indirect expression of magnitude) | It is set approximated (e.g., as a linear model in the form of Taylor series) | Regression model on the results of implementation of the measurement experiment plan | Other |

| 3 Determination of input values | Differential approach. The standard uncertainties of each input quantity are determined separately by type A or B | Comprehensive approach. The total standard uncertainty of the measurement result is determined at once |
|--------------------------------|---------------------------------------------------------------|---------------------------------------------------------------------------------
|                                  | The distribution law is not used (frequency approach)          | The law of distribution is used (Bayesian approach) |
| 4. Transformation of distribution laws | GUM method Empirical methods Combined method | Monte Carlo method Other |
Obviously, if the uncertainty assessment process is composite, it is important to know the advantages and disadvantages of each element in order to design the most appropriate uncertainty assessment process for the laboratory environment.

It should be noted that the morphological box has an open architecture. In this sense, it is of scientific interest as a tool for generating new methods at different stages of the uncertainty assessment process. In Table 1, their place is highlighted as "Other" on a gray background. We can give a number of examples of developments of the authors of the article concerning new methods of realization of separate stages of the uncertainty assessment process:

Stage 1 (see Table 1). Identification of influencing factors. Methodology of identification of input values of the measurement process using IDEF0 functional modeling methodology and the method of alternatives as a method of expert evaluation [7, 12].

Stage 2 (see Table 1). Formation of the measurement model. To solve complex measurement problems of data analysis and decision making under conditions of significant a priori uncertainty, in our opinion, methods of nonparametric estimation of passively collected measurement information, for example, based on wavelet transforms, are promising [20]. The use of such models for solving this class of problems allows minimizing the methodological error of linear approximation.

Stage 4 (see Table 1). Transformation of distribution laws. As an alternative to the Monte Carlo method, we can propose the author’s "method of successive transformations" (MPT), which is designed for efficient uncertainty estimation under a given mathematical model of measurements (1) with an arbitrary number of input quantities and a single output quantity [2]. The efficiency of the estimation is manifested in the fact that MPT realizes the algorithm of estimation of measurement uncertainty through transformation of distributions of input quantities. In this case, in contrast to the Monte Carlo method, the MPT assumes the possibility of a typical solution of the estimation problem with minimum resource consumption. The key point of the proposed method is the technique of pairwise analytical convolution of distributions of input quantities taking into account the measurement model $Y = f(X_1, \ldots, X_N)$.

Table 2

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<th>Element (stage of the evaluation process)</th>
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<td>Each input quantity is not defined separately</td>
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<td>4. Transforming distribution laws</td>
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Options for implementing the method uncertainty assessment process in the laboratory

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Conclusion

It can be stated that the validity of measurement uncertainty estimation is provided from the position of the system approach through the development of the estimation process as an algorithm of actions. Taking into account the fact that different stages of the process of measurement uncertainty estimation can be realized by alternative methods, the possibility of developing the most acceptable algorithm of actions for the conditions of the measurement laboratory is determined. To solve the problems of determining the full set of variants of realization of each stage of the process of uncertainty estimation for specific conditions the morphological box method is proposed. The structure of the morphological box of a typical uncertainty estimation process with an open architecture, which is based only on the generally accepted methods and approaches of realization of each stage of the process, is substantiated. The possibilities of application of the morphological box method for development of an effective algorithm of the process of uncertainty estimation of the measurement method in the laboratory and for improvement of the estimation process in terms of creation of new methods of realization of separate stages of the process are demonstrated by examples.

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