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Minimising the Influence of the Distribution Law on the Reliability of Accuracy Estimation

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Abstract

The problem of minimising the influence of distribution laws of input values on the reliability of results in evaluation models in the field of metrology is considered. Aim of this work was to substantiate rational approaches and methods of correct solution of the problem in the case when the distribution law of input values differs from normal. Classification of variants of solutions to the problem of normality of input values in models of estimation of uncertainty of measurement method, metrological reliability of measuring instrument, etc. is presented. The complex problem of estimating the law of input quantity distribution and bringing it to normal by correcting its probabilistic characteristics is formulated. It is substantiated that such a solution of the problem will provide 'frequency equivalence' of empirical and normal distribution laws. Methods of solving the problem for two possible cases are considered: the input values of the model are estimated a priori and empirically. The variants of the rational solution of the problem for the case of a priori estimation of the input value of the model (type B), generally accepted in metrological practice, are considered. The main attention is paid to the case of estimating the input value of the model empirically (by type A). Chebyshev's and Vysochansky-Petunin's inequalities are taken as theoretical prerequisites for solving the problem which determine the estimates from above of the probability of deviation of a random variable from the mean without taking into account the exact form of its distribution law. A graphical method of estimating the 'degree of normality' of the empirical law of distribution of an input quantity and bringing it to normal by correcting its statistics is proposed. Implementation of the method assumes use of statistical packages of applied programs, for example, Statistica package, and visual comparison of the histogram of empirical distribution with the theoretical curve of normal distribution. For all possible situations an algorithm of actions is defined including analyses of the degree of mismatch between distributions and decisive rules for correcting the initial statistics of the input quantity.

Keywords: estimation models, input quantities, empirical distribution law, normalization to Gaussian distribu-tion

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Минимизация влияния закона распределения величины на достоверность оценивания точности

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Рассмотрена задача минимизации влияния законов распределения входных величин на достоверность результатов в моделях оценивания в области метрологии. Целью данной работы являлось обоснование рациональных подходов и методов корректного решения задачи в случае, если закон распределения входных величин отличен от нормального. Представлена классификация вариантов решений проблемы нормальности входных величин в моделях оценивания неопределённости метода измерений, метрологической надёжности средства измерений и др. Сформулирована комплексная задача оценивания закона распределения входной величины и приведения его к нормальному путём корректирования её вероятностных характеристик. Обосновано, что подобное решение задачи позволит обеспечить «частотную эквивалентность» эмпирического и нормального закона распределения. Рассмотрены способы решения задачи для двух возможных случаев: входные величины модели оцениваются априори и эмпирически. Рассмотрены общепринятые в метрологической практике варианты рационального решения задачи для случая оценивания входной величины модели априори (по типу Б). Основное внимание уделено случаю оценивания входной величины модели эмпирически (по типу А). В качестве теоретических предпосылок решения задачи приняты неравенства Чебышева и Высочанского–Петунина, которые определяют оценки сверху вероятности отклонения случайной величины от среднего без учёта точной формы её закона распределения. Предложен графический метод оценки «степени нормальности» эмпирического закона распределения входной величины и приведения его к нормальному путём корректирования её статистик. Реализация метода предполагает использование статистических пакетов прикладных программ, например, пакета Statistica, и визуальное сравнение гистограммы эмпирического распределения с теоретической кривой нормального распределения. Для всех возможных ситуаций определён алгоритм действий, включающий анализ степени несоответствия распределений и решающие правила в отношении корректирования исходных статистик входной величины.

Ключевые слова: модели оценивания, входные величины, эмпирический закон распределения входной величины, приведение к нормальному закону распределения

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Introduction

All estimation models in metrology rely on probability theory and mathematical statistics: evaluating instrument errors, calibration intervals, bias, precision, measurement uncertainty, laboratory proficiency testing, etc. [1–5].

The calculated parameter y is treated as an integrated random variable resulting from combining influencing random variables x_i. According to the Central Limit Theorem, normality of input distributions is critical. For normally distributed x_i, the result y is also normally distributed:

– expected value y^0 is determined as:

$$y^0 = f(m(x_1), m(x_2), \ldots, m(x_N)), \tag{1}$$

where f is the mathematical model; $m(x_i)$ are the expected values of inputs, $i = 1\ldots N$;

– is the standard deviation $\sigma(y)$ of the design parameter y, defined as:

$$\sigma(y) = \sqrt{\sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i}\right)^2 \sigma^2(x_i)}, \tag{2}$$

where $\sigma(x_i)$ are the standard deviations of the input quantities.

The risk of incorrect definition of the distribution law of the input quantity x_i has a hidden influence on the estimates of y_0 and $\sigma(y)$. For example, if the distribution law for the input quantity x_i is defined as normal, while the true (unknown) law of its distribution is equal probability, the error in defining $\sigma(x_i)$ in formula (2) will be up to 70 %. This, in turn, will lead, respectively, to the error in determining $\sigma(y)$ [3–8].

Obviously, the problem of normality of input values x_i of any estimation model includes the solution of two problems (Figure 1):

– Task 1. The problem of assessing the conformity of the distribution law of the value x_i to the normal law [7–10].

– Task 2. The problem of ensuring the possibility of correct use of probabilistic characteristics $m(x_i)$ and $\sigma(x_i)$ of the quantity x_i in the estimation models (1) and (2) if its distribution law differs from the normal law [11–14].

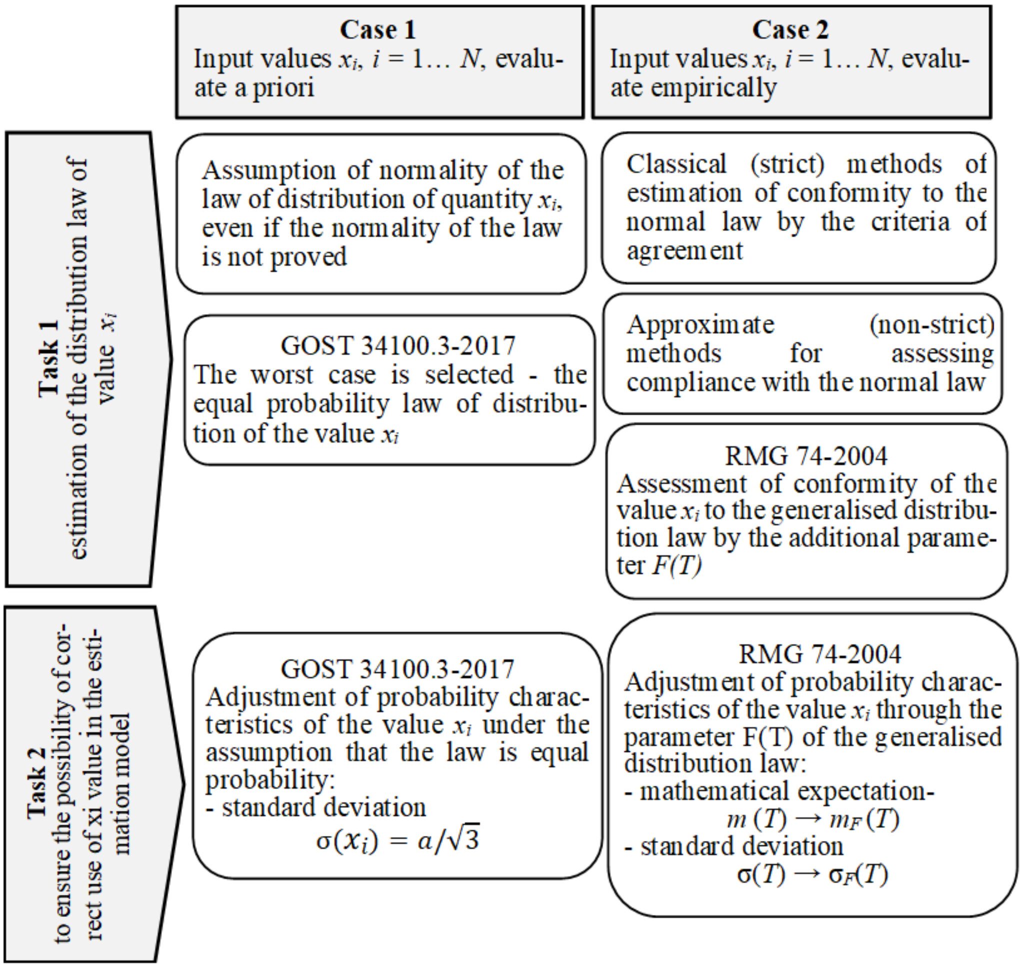

Figure 1 – Classification of variants of solutions to the problem of normality of input values xi in estimation models

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The present study focuses on solving these problems.

Methodology for tackling the defined tasks

It is obvious that the best solution of these two problems is a complex solution. Ultimately, it is necessary to form and correctly solve equations (1) and (2) even if the input values x_i are not normally distributed. From this point of view, it is rational to combine these two problems into one complex problem of estimating the distribution law of the input quantity x_i and bringing it to the normal law.

As follows from Figure 1, the problem of normality of the law of distribution of the value x_i in applied problems, including the problems of applied metrology, is solved by bringing the known or assumed empirical distribution of the input value x_i to the normal distribution by correcting in one way or another the probabilistic characteristics – mathematical expectation $m(x_i)$ and standard deviation $\sigma(x_i)$. This ensures 'frequency equivalence' of the obtained working model of estimation and the model in which the same input values x_i would be distributed according to Gaussian law.

Note. The notion of 'frequency approach' to estimation of uncertainty of measurements is in-troduced by standard[1]. The GUM method[2] belongs to this type of approach [3, 4].

For the purposes of the analysis, it is rational to identify and consider two main cases and their corresponding approaches and methods minimising the influence of the distribution law of x_i on the reliability of estimation models (1) and (2) (Figure 1).

Case 1. Input values x_i $i = 1...N$, are estimated a priori.

In this case, the best metrological practice considers two options for solving the problem.

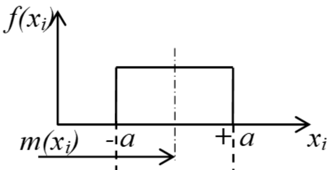
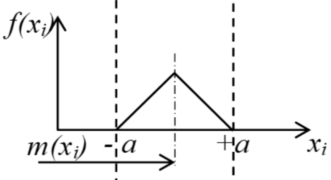
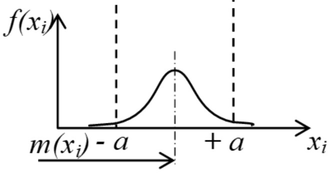
1. The assumption of normality of the law of distribution of possible values of the measured quantity is made, even if the normality of the law is not proved.

2. The GUM method recommends to consider the empirical distribution law close to the equiprobable one and to accept the corrected value of the standard deviation as in Table 1 when there are difficulties in determining the law of distribution of the input quantity x_i.

The correction formulas are valid provided that the assumed spread $2a$ of x_i is the same in all cases.

Table 1

Adjustment of standard deviation values taking into account the law of distri-bution of a random variable

Law of distribution of the input quantity x_i	Graphical representation of the distribution cone (scope $2a$ = Const)	Adjusted standard deviation $\sigma(x_i)^{corr}$	Adjustment coefficient
Equal probability		$a/\sqrt{3}$	1.7
Triangular (Simpson)		$a/\sqrt{6}$	1.22
Normal (Gaussian)		$a/3$	1.0

[1]GOST 34100.1-2017/ ISO/IEC Guide 98-1/ 2009 Uncertainty of measurement – Part 1: Introduction

[2]GOST 34100.3/ ISO/IEC Guide 98-3:2008, Uncertainty of measurement – Part 3: Guide to the expression of uncertainty in measurement (GUM:1995)

There is an obvious pattern: the more the distribution law of the input quantity differs from the normal distribution law, the more the standard deviation $\sigma(x_i)^{corr}$ is corrected (upwards) for the same assumed magnitude. This is evidenced by the values of the adjustment coefficients. This is logical as it reduces the risk of underestimating the result in the estimation model (2) and hence the risk of the consumer.

Case 2. Input values x_i, $i = 1...N$, are estimated empirically.

The availability of an array of experimental data x_{ij}, $j = 1…k$, where k is the size of the array, allows us to determine a number of statistics that can contribute to an acceptable estimation of the distribution law of value x_i: mathematical expectation $m(x_i)$, standard deviation $\sigma(x_i)$ or its estimate $s(x_i)$, histogram, etc.

To solve problem 1, a generally recognised evidentiary approach is the application of classical (strict) methods of assessing compliance with the normal law by the criteria of agreement [5]. A number of factors determines the choice of the criterion of agreement: the size of the array of values.

Alternative to the criterion approach are methods of approximate estimation of normality of the law of distribution of value x_i. The most common among them are:

1. Estimation method using the coefficient of variation. The coefficient of variation v of the random variable x_i is defined as $v = \sigma(x_i)/m(x_i)$.

From the experience of statistical studies of technological processes, it is known that if $v<0.33$, then there is a high probability that the random variable x_i obeys the normal distribution law [6].

2. Graphical method of estimating the law of distribution of the value x_i. It is based on the visual method of comparing the histogram of the empirical distribution of the values of the array x_{ij} with the theoretical normal distribution curve. The histogram allows us to qualitatively assess not only the degree of non-compliance of the empirical distribution law with the normal distribution law, but also to visually determine the form of the empirical law (triangular, equal probability, etc.).

It should be noted that the above methods of assessing the conformity of the distribution of the input quantity to the normal law by both strict (according to the criteria of agreement) and non-strict methods do not allow us to solve problem 2 – the problem of bringing the distribution law of a quantity to the normal law by correcting the probabilistic characteristics of x_i ($m(x_i)$ and $\sigma(x_i)$).

From this point of view, the approach outlined in document[3] is of interest. In order to ensure the correctness of the regression model of the drift of metrological characteristics of measuring instruments, this document introduces the notion of a generalized law of distribution with respect to input quantities and proposes a peculiar mechanism of their ‘frequency’ correction if they are not distributed according to the normal law.

The mechanism of correction of the regression model of drift of metrological characteristics of measuring instruments, as a peculiar mechanism of replacement of variables, assumes the introduction into the expression of the generalized law of distribution of a random variable of the additional parameter $F(T)$, the value of which is individual for each case [2, 4].

Note. For $F(T) = 1$, the generalised distribution law takes the form of Gaussian normal law.

The variables are replaced as follows:

– the mathematical expectation $m(T)$ of errors x_i of measuring instruments in the controlled lot is replaced by $m_F(T)$;

– standard deviation $\sigma(T)$ of errors x_i of measuring instruments in the controlled lot is replaced by σ_F(T);

– tolerance limit of measurement instruments Δ is replaced by $\Delta^{F(T)}$.

The corrected values ($\Delta^{F(T)}$, $\mathrm{m}_F^{(T)}$, $\sigma_F^{(T)}$ are substituted into the drift equations of the type (1) and (2) [2, 4].

It should be noted that the procedure of finding the parameter$F(T)$ of the generalised distribution law is rather time-consuming, and the value of the parameter is determined by the criterion of maximum likelihood by enumerating the matrix of F values from 0 to 4 with a step of 0.1, moreover for each verification that preceded the current moment for the investigated type of measuring instruments. The sought parameter $F(T)$ provides ‘equivalence’ of the drift equations to the conditions as if the model parameters Δ, $m(T)$, $\sigma(T)$ were distributed according to the Gaussian law.

The above arguments allow us to suggest that for the correctness of expressions (1) and (2) it is not so much the fact of normality of the distribution laws of input quantities that is important, as the fact of

[3]RMG 74-2004 State system of ensuring uniformity of measurements. Methods of determination of interverification and intercalibration intervals of measurements

correctness of redefining (correcting) their probability characteristics $m(x_i)^{corr}$ and $\sigma(x_i)^{corr}$ depending on the form of the empirical distribution law ('degree of normality') for substitution into the estimation models (1) and (2).

Obviously, this complex problem is relevant, especially for small samples or asymmetric distributions, where the exact form of the law is unknown, but it is necessary to guarantee the reliability of estimates at the levels of $P = 0.95$ or $P = 0.99$.

Theoretical prerequisites for solving the problem

First, the Chebyshev's inequality, according to which a random variable mainly takes values close to its mean and which limits the probability of large deviations of a random variable from its mathematical expectation, testifies in favour of the formulated assumption:

$$P(|X - m(x_i)| \geq k\,\sigma(x_i)) \leq 1/k^2, \quad (3)$$

where k – coefficient [7, 8].

Chebyshev's inequality gives an estimate from above of the probability of a random variable deviating from the mean, without requiring knowledge of a particular distribution, but subject to finite variance.

The Chebyshev random variable fits within the range $\pm 2\sigma(x_i)$ with probability $P = 0.75$ and within the range $\pm 3\sigma(x_i)$ with probability $P = 0.89$.

The arguments in favor of the formulated assumption are strengthened by the Vysochansky–Petunin inequality, according to which the random variable fits into the range $\pm 2\sigma(x_i)$ with probability $P = 0.89$ and within the range $\pm 3\sigma(x_i)$ with probability $P = 0.95$.

$$[P\,(|X - m(x_i)| \geq \lambda\,\sigma(x_i))] \leq 4/9\lambda^2, \quad (4)$$

where λ – coefficient [7–10].

The Vysochansky–Petunin inequality gives an improved estimate of the probability of de-viation of a random variable from the mean compared to the Chebyshev inequality, if the specific distribution is unimodal ('peak-shaped') and the dispersion is finite. The ratios of confidence limits of distributions for different probabilities P are presented in Table 2.

Table 2

Adjustment of standard deviation values taking into account the law of distri-bution of a random variable

N	Confidence probability, P	Confidence limits for the probability P		
		Gaussian distribution	Arbitrary unimodal distribution (Vysochansky–Petunin inequality)	Arbitrary distribution (Chebyshev inequality)
1	0.90	$\pm 1.64\,\sigma(x_i)$	$\pm 2.11\,\sigma(x_i)$	$\pm 3.16\,\sigma(x_i)$
2	0.954	$\pm 2.0\,\sigma(x_i)$	$\pm 2.98\,\sigma(x_i)$	$\pm 4.47\,\sigma(x_i)$
3	0.997	$\pm 3.0\,\sigma(x_i)$	$\pm 12.17\,\sigma(x_i)$	$\pm 18.26\,\sigma(x_i)$

We propose a method for estimating the 'degree of normality' of the distribution law of the input quantity x_i and bringing its distribution law to normal by correcting its statistics ($m(x_i)$ and $\sigma(x_i)$). The aim is correct application in estimation models (1) and (2). The method is based on:

– graphical representation of the empirical distribution (histogram);

– subsequent analysis of the empirical and normal distributions;

– decision making on 'frequency' correction of probabilistic characteristics $m(x_i)$ and $\sigma(x_i)$ of the input quantity.

The method involves the use of statistical packages of application programmes, for example, the Statistica package [11, 12].

The graphical method of estimating the distribution law of x_i is based on the visual method of comparing the histogram of the empirical distribution of x_{ij} array values with the theoretical normal distribution curve, such that $s(x_i) = \sigma^{teor}$ (Figure 2) [13].

Note. The Statistica package automatically superimposes a normal distribution curve with a standard deviation equal in numerical value to the histogram of the empirical distribution.

In view of the above, we can formulate a criterion for assessing the 'degree of normality' of the empirical distribution law of the value x_i and a decisive rule for bringing the distribution law of the value to normal by correcting its probability characteristics ($m(x_i)$ and $\sigma(x_i)$):

«If the histogram of the empirical distribution of the input quantity x_i fits within the limits $\pm 2\sigma^{teor}$ (P = 0.95) or $\pm 3\sigma^{teor}$ (P = 0.99)) of the normal distribution law, and $\sigma(x_i) = \sigma^{teor}$, the hypothesis of equivalence of the empirical law to the normal law can be accepted and the values of m^{teor} and σ^{teor} can be correctly substituted into the expressions of the estimation models (1) and (2)».

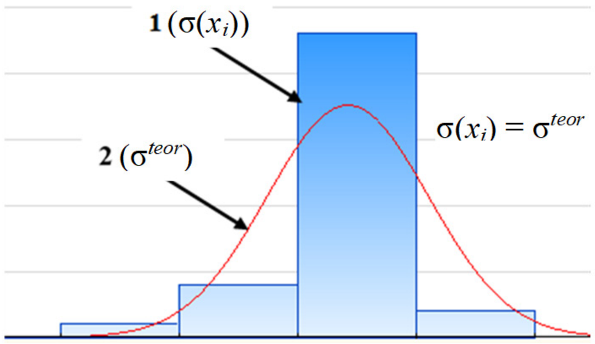


Figure 2 – Graphical representation in Statistica of the histogram of the empirical distribution (1) and the theoretical curve of the normal distribution (2) under the condition of equality of standard deviations.: $\sigma(x_i) = \sigma^{teor}$

The algorithm of estimation of the empirical law of distribution of the input quantity and its reduction to normal is as follows:

1. An experimental study of the input quantity x_i is carried out in accordance with the established plan. The experiment plan and conditions of its realization are determined by the specific task.

For example, for the task of estimation of uncertainty of measurement method – the plan of estimation of standard uncertainty $u(x_i)$ of input quantity xi by type *A* [14, 15]. For the task of determining the interval between the verification intervals of measuring instruments using the drift model – the plan of investigation of metrological characteristics of the controlled batch [16].

2. An array of statistical data on the input quantity x_i is formed, which is analyzed for the presence of outliers and scatterings and subjected to the necessary correction [17].

3. Using the Statistica application package, a histogram of the empirical distribution of the value and a normal distribution curve automatically superimposed on it with standard deviations equal in numerical value $\sigma(x_i) = \sigma^{teor}$ (Figure 3).

4. We visually analyse the degree of correspondence between the empirical and normal distributions of the input quantity x_i. Three situations are possible here.

Situation 1. The histogram of the empirical distribution of the input quantity x_i on both sides does not exceed the confidence limits $\pm 2\sigma^{teor}$ (P = 0.95) or $\pm 3\sigma^{teor}$ (P = 0.99) of the theoretical normal distribution curve.

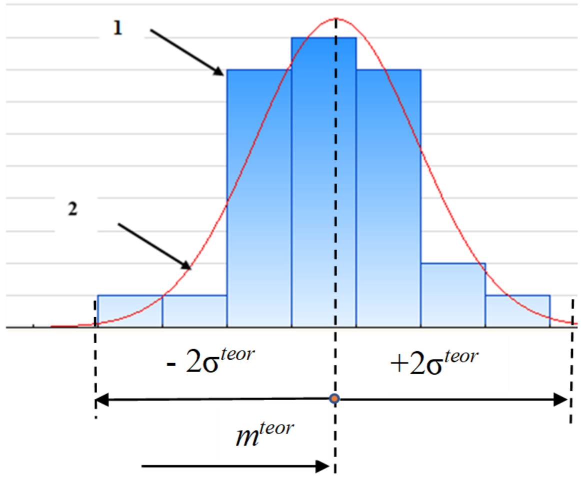


Figure 3 – Situation 1: Mutual location of the histogram of the empirical distribution (1) and the theoretical curve of the normal distribution (2) under the condition of equality of standard deviations: $\sigma(x_i) = \sigma^{teor}$

Decisive rule: for this case the formulated criterion can be used with high confidence and the values of m^{teor} and σ^{teor} can be correctly substituted into the expressions of the estimation models (1) and (2).

Situation 2. The histogram of the empirical distribution of the input quantity x_i on one (Figure 4*a*) or both sides (Figure 4*b*) falls outside the $\pm 2\sigma$ (P = 0.95) or $\pm 3\sigma$ (P = 0.99) confidence limits of the theoretical normal distribution curve. This fact indicates the non-equivalence of the empirical and theoretical distribution laws, despite the numerical equality of standard deviations $\sigma(x_i) = \sigma^{teor}$.

Decisive rule: for this case one should make a decision guided by Chebyshev's (3) and Vysochansky–Petunin's (4) inequalities:

– if the histogram of the empirical distribution of the input quantity has a pronounced unimodal form, then the correction of the standard deviation $\sigma(x_i)$ of the quantity x_i for fitting into the estimation model (2) should be made on the basis of the Vysochansky–Petunin inequality (4) in accordance with Table 3.

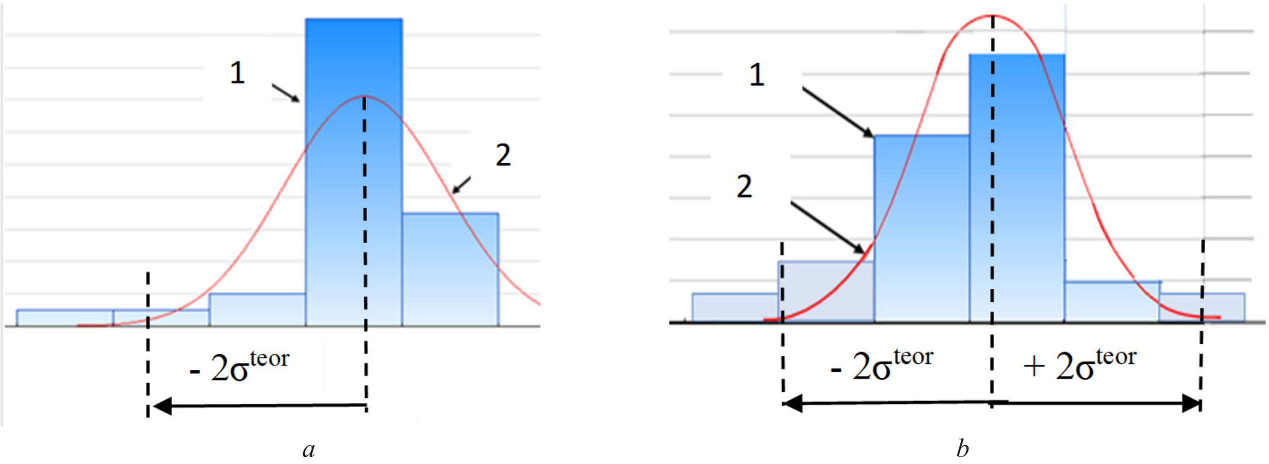


Figure 4 – Situation 2: Mutual location of the histogram of the empirical distribution (1) and the theoretical normal distribution curve (2) under the condition of equal standard deviations: $\sigma(x_i) = \sigma^{teor}$; *a* – the empirical distribution is asymmetric; *b* – the empirical distribution is symmetric

Table 3

Correction of the standard deviation σ(x_i) depending on the probability *P* (the Vysochansky–Petunin inequality)

Confidence probability P	Probabilistic characteristics of empirical and theoretical normal distributions of a value x_i	Adjusted value $\sigma(x_i)$ of the empirical distribution x_i
$P = 0.9$		$\sigma(x_i)^{corr} = (2.11 / 1.64)\sigma^{teor} = 1.29\sigma^{teor}$
$P = 0.95$	$\sigma(x_i) = \sigma^{teor}$	$\sigma(x_i)^{corr} = (2.981 / 2)\sigma^{teor} = 1.49\sigma^{teor}$
$P = 0.9973$		$\sigma(x_i)^{corr} = (12.17 / 3)\sigma^{teor} = 4.06\sigma^{teor}$

Note. $\sigma(x_i)^{corr}$ are determined based on the data in Table 2.

– If the histogram of the empirical distribution of the input quantity does not have an explicit unimodal form, the calculation of the standard deviation of $\sigma(x_i)$ for fitting into the estimation model (2) should be performed on the basis of Chebyshev's inequality (3) in accordance with Table 4.

Table 4

Correction of the standard deviation σ(x_i) depending on the probability *P* (the Chebyshev's inequality)

Confidence probability P	Probabilistic characteristics of empirical and theoretical normal distributions of a value x_i	Adjusted value $\sigma(x_i)$ of the empirical distribution x_i
$P = 0.9$		$\sigma(x_i)^{corr} = (3.16/ 1.64)\sigma^{teor} = 1.93\sigma^{teor}$
$P = 0.95$	$\sigma(x_i) = \sigma^{teor}$	$\sigma(x_i)^{corr} = (4.47 / 2)\sigma^{teor} = 2.24\sigma^{teor}$
$P = 0.9973$		$\sigma(x_i)^{corr} = (18.26/ 3)\sigma^{teor} = 6.09\sigma^{teor}$

Note. $\sigma(x_i)^{corr}$ determined on the basis of the data in Table 2.

Thus, the proposed algorithm for estimating the empirical distribution law of the input random variable and converting it to a normal distribution covers all possible scenarios regarding the mutual arrangement of the empirical distribution histogram and the theoretical normal distribution curve, ensuring an unconditional solution to the problem.

Conclusion

The problem of normality of the law of distribution of input quantities x_i in estimation models in the problems of applied metrology is considered. Two main cases and corresponding approaches and methods minimising the influence of the distribution law of x_i on the reliability of estimation models are identified: input values x_i, $i = 1...N$, are estimated a priori (by type *B*) or empirically (by type (*A*)). It is suggested that for the correctness of estimation models it is important not so much the fact of normality of the distribution law of the input quantity x_i as the fact of estimating the form of its empirical distribution law (‘degree of normality’) and correctly redefining (correcting) its probabilistic characteristics $m(x_i)^{corr}$ and $\sigma(x_i)^{corr}$ for substitution in the estimation model. Chebyshev's inequality and Vysochansky–Petunin inequality are presented as theoretical prerequisites for solving the problem, which define estimates from above of the probability of deviation of a random variable from the mean without taking into account the exact form of its distribution law. A graphical method of estimating the ‘degree of normality’ of the distribution law of the input quantity x_i and correcting its statistics is proposed. The method implies the use of statistical packages of application programs, for example, the Statistica package. All possible situations with mutual location of the histogram of empirical distribution and theoretical curve of normal distribution of input quantity x_i under the condition of equality of standard deviations are considered: $\sigma(x_i) = \sigma^{teor}$. For each situation, we formulate a decisive rule that determines the corrected value of the standard deviation $\sigma(x_i)^{\mathrm{corr}}$, which can be correctly used in uncertainty estimation models.

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