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Modeling of Passive Q-Switched Yb, Er Glass Lasers with Transverse Pumping by Linear Laser Diode Arrays

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Abstract

A mathematical model of a passively Q-switched solid-state laser based on ytterbium and erbium co-doped active media with transverse pumping by linear laser diode arrays has been developed. The gain in the laser is calculated using rate equations taking into account the space-time dependence of the pump radiation intensity in the laser element. The output laser characteristics, the pulse energy, the peak pulse power and the pulse duration, are calculated using analytical equations obtained for a passively Q-switched solid-state laser in the approximation of a "slow" saturable absorber. The model allows one to find the range of parameters of the active element, passive modulator, resonator, and pumping system at which the generation threshold is reached and the laser generates pulses with the required energy and duration. Modeling results were used for the experimental development of lasers with an active element based on phosphate glass doped by ytterbium and erbium ions. The energy and duration of the output light pulses were ≈1 mJ, ≈40 ns, and ≈2 mJ and ≈20 ns, depending on the content of ytterbium and erbium ions in the active element, as well as on the initial transmission of the passive modulator and the resonator parameters.

Keywords: mathematical model of a passive Q-switched solid-state laser, Yb^{3+} and Er^{3+} ions co-doped phosphate glasses, transverse laser pumping by using linear laser diode arrays

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Моделирование лазеров на основе иттербий-эрбиевого стекла с пассивной модуляцией добротности при поперечной накачке линейками лазерных диодов

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Разработана математическая модель твердотельного лазера с пассивной модуляцией добротности на основе активных сред со-легированных иттербием и эрбием, с поперечной накачкой линейками лазерных диодов. Усиление в лазере рассчитывается с использованием скоростных уравнений с учётом пространственно-временной зависимости интенсивности излучения накачки в лазерном элементе. Выходные характеристики лазера, энергия импульса, пиковая мощность импульса и его длительность, рассчитываются с использованием аналитических уравнений, полученных для твердотельного лазера с пассивной модуляцией добротности в приближении «медленного» насыщающегося поглотителя. Модель позволяет найти диапазон параметров активного элемента, пассивного модулятора, резонатора и системы накачки, при которых достигается порог генерации и лазер генерирует импульсы с требуемой энергией и длительностью. Результаты моделирования использованы для экспериментальной разработки лазеров с активным элементом на основе фосфатного стекла, со-активированного ионами иттербия и эрбия. Энергия и длительность выходных световых импульсов составляли ≈1 мДж, ≈40 нс и ≈2 мДж и ≈20 нс в зависимости от содержания ионов иттербия и эрбия в активном элементе, а также от начального пропускания пассивного модулятора и параметров резонатора.

Ключевые слова: математическая модель твердотельного лазера с пассивной модуляцией добротности, фосфатные стекла, со-активированные ионами Yb^{3+} and Er^{3+}, поперечная накачка лазеров линейками лазерных диодов

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Introduction

Lasers based on ytterbium- and erbium- doped glasses generating nanosecond light pulses in the spectral range of 1.5 μm are of great interest for ranging, see, e.g. [1–7]. To obtain pulses with an energy of several millijoules, a transverse pumping scheme is often used with radiation from flash-lamps or laser diode arrays (LDAs). Currently, there are a number of commercially available LDAs with radiation power of several tens of watts in a spectral range matched with the absorption lines of ytterbium ions. Use of LDAs allows for reduction in the thermal load on the active element, an increase in the durability of laser operation, and creation of a more compact laser design compared to lamp analogs.

Ytterbium ions, which have intense absorption lines in the 940 nm region in phosphate glasses, absorb pump radiation and nonradiatively transfer excitation energy to erbium ions according to the scheme $^2F_{5/2}(Yb^{3+}) + {}^4I_{15/2}(Er^{3+}) \rightarrow {}^2F_{7/2}(Yb^{3+}) + {}^4I_{11/2}(Er^{3+})$, see Figure 1. Fast relaxation (a few microseconds) of erbium ions to the upper laser level $^4I_{11/2} \rightarrow {}^4I_{13/2}$ allows to reduce losses due to up-conversion from the $^4I_{11/2}$ level.

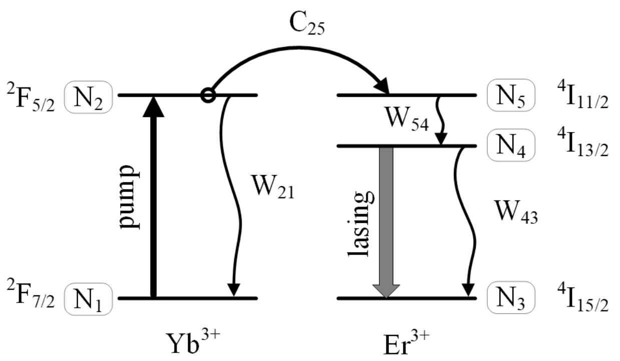

Figure 1 – Scheme of energy levels of Yb^{3+} and Er^{3+} ions involved in the process of pumping $^2F_{7/2}\rightarrow{}^2F_{5/2}$ and lasing $^4I_{13/2}\rightarrow{}^4I_{15/2}$ as well as the scheme of the excitation energy transfer $^2F_{5/2}$ $(Yb^{3+})\rightarrow{}^4I_{11/2}$ (Er^{3+}). N_α is the population of the α-th energy level (α = 1, 2, 3, 4, 5)

$Co^{2+}:MgAl_2O_4$ (Co:MALO) crystals are widely used in solid-state lasers with passive Q-switching in the 1.5 μm spectral region as materials for saturable absorbers. Technology for their growth is well developed, and the spectroscopic characteristics have been studied in detail [8]. Therefore, for lasers under consideration, thin plane-parallel plates of Co:MALO were used as passive modulators.

Efficiency of a transverse pumped laser depends largely on matching of the volume of the lasing mode and the volume of the active medium in which the gain is created using pump radiation. As a rule, the laser cavity is designed so that the spatial distribution of the lasing intensity is close to the TEM_{00} mode. As for the spatial distribution of the pump radiation in the active medium, the situation is more complicated. This distribution depends significantly on the relative position of the active element and the LDAs, geometry of the active element, presence of a focusing system, and other factors. Many works are devoted to studying characteristics of transverse pumped solid-state ytterbium-erbium lasers and optimizing their design, see, e.g. [3, 9–18].

One of the important areas of research into such lasers is mathematical modeling of their operation. In [14], characteristics of a passive Q-switched laser based on ytterbium-erbium glass pumped by linear LDAs were studied. The results of numerical modeling of spatial distribution of the density of the absorbed pump radiation power over the active element at different distances between the active element and LDAs are presented. In [15] and [16], mathematical models of passive Q-switched lasers based on a system of rate equations that take into account up-conversion transitions in erbium ions and the presence of "unpumped" regions in the active element were proposed. In [15], overlap of the volume in the active medium where the gain has been created with the volume of the generated radiation is taken into account in the absorbed power of the pump radiation. In a more complete model [16], the spatial distribution of the inverse population at the laser transition of erbium ions was calculated in the geometric optics approximation without focusing the radiation of linear LDAs onto the surface of the active element. Then, the inverse population was averaged over the cross-sectional area of the generated radiation mode. Active elements in the form of rectangular parallelepipeds and cylinders were considered.

In this paper, we use an approach to modeling the operation of ytterbium-erbium glass lasers with transverse pumping by linear LDAs radiation, based on the results of [19], where a number of analytical expressions were obtained for calculating the output characteristics of a passive Q-switched laser based on a quasi-three-level active medium with a "slow" saturable absorber. Here it is assumed that the relaxation time of the passive modulator, τ_{sa}, is much longer than the duration of the generated pulses, τ_l, which is quite true in our case, $\tau_{sa} \approx 300$ ns [8] and $\tau_l \approx 10 \div 50$ ns. Within the framework of the system of rate equations with parameters depending on spatial coordinates, the time dependence of the average-weighted gain coefficient by the mode volume of the generated radiation is calculated. This allows us to check the fulfillment of threshold conditions in the laser and to determine the time delay between the start of the pump pulse and the start of the generation pulse.

2. Laser model

2.1. Scheme of the laser with transversal pumping

Scheme of a passive Q-switched, transverse pumped laser is shown in Figure 2. The generated laser radiation propagates along the Y axis, the pump radiation propagates along the Z axis.

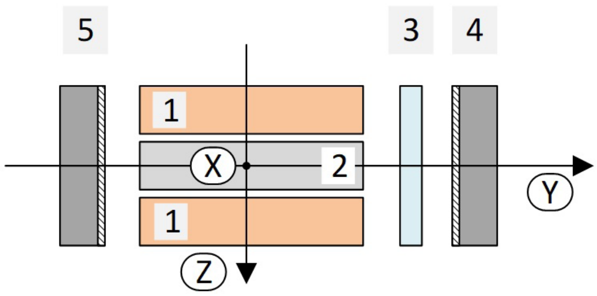

Figure 2 – Scheme of the passive Q-switched laser: 1 – the linear LDA; 2 – the active element; 3 – the passive Co:MALO modulator; 4 and 5 – the OC and HR mirrors of the resonator, respectively

Transverse pumping is carried out by uncollimated radiation beams of two linear LDAs, see Figure 3. This pumping scheme seems simpler from the point of view of practical implementation, since it does not require additional collimating optical elements.

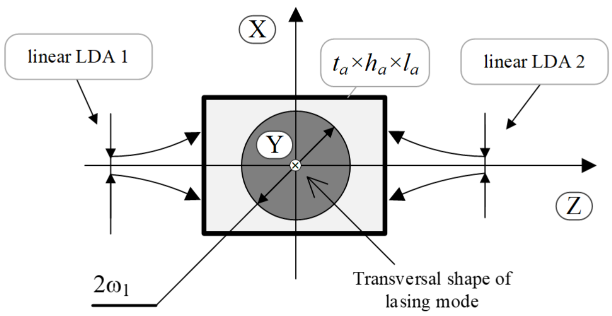

Figure 3 – General scheme of pumping without collimation of radiation of linear LDAs. t_a, h_a and l_a are the width, height and length of the active element, respectively, $2\omega_l$ is the Gaussian diameter of the fundamental mode of the generated radiation at the level of $1/e^2$

As a source of optical pumping, we will consider a linear LDA in the form of a strip with dimensions of 1 μm × 10 mm, emitting light at a wavelength of λ_p. The spatial characteristics of such light sources are characterized by the so-called "fast" and "slow" axes. The "fast" axis is directed perpendicular to the emitting strip, in the direction of this axis the divergence of the light beam is determined by diffraction on its aperture. Along the "slow" axis, which is directed along the emitting strip, the spatial characteristics of the pump beam are described in the approximation of geometric optics. In future calculations, the divergence of light along this "slow" axis will not be taken into account.

Calculation of the spatial distribution of pump radiation in the volume of the active medium at a certain point in time is carried out under the following assumptions:

– pump radiation beam in the XZ plane is Gaussian-like, the optical quality of the beam is described by the parameter M^2;

– shape of the distribution of the pump beam intensity as it propagates along the Z axis remains unchanged;

– width $\omega_p(z)$ of the pump beam along the Z axis is calculated using the ABCD method [20].

2.2. Basic equations and formulas for calculating the gain coefficient

To perform numerical calculations a grid of spatial coordinates is constructed in the volume of the active element by dividing it into planes parallel to the Y axis and perpendicular to the X and Z axes, see Figure 4, $[x, z]_{i,j} = x_i, z_j$, $i = 1 \ldots N_x$, $j = 1 \ldots N_z$. Here N_x and N_z are the number of partitions into layers perpendicular to the X and Z axes, respectively.

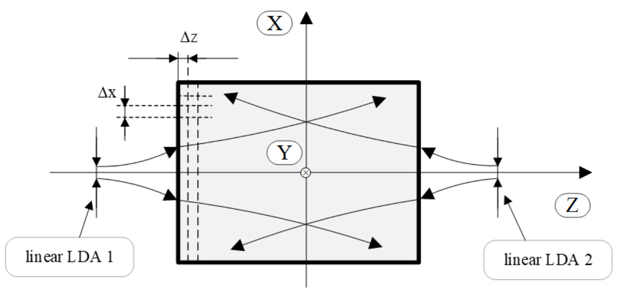

Figure 4 – Pumping scheme without collimation of radiation from linear LDAs with partition of the active element into layers parallel to the Y axis and perpendicular to the X and Z axes

Next, a time grid is constructed as a set of time moments t_k, separated by time intervals Δt,

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$k = 1...N_t$, where N_t is the number of partitions of the pump pulse of duration t_p. The value of Δt is much less than the relaxation time of energy levels of ytterbium and erbium ions. The space-time grid allows one to introduce arrays of energy level populations, see Figure 1, the elements of which can be written as $[N_\alpha]_{i,j,k} = N_\alpha(x_i, z_j, t_k)$, ($\alpha$ = 1, 2, 3, 4, 5).

Let us briefly describe the algorithm for calculating the average-weighted gain coefficient by the volume of the lasing mode at different moments of time $\bar{k}_l(t_k) = [k_l]_k$ at the pumping stage. When the pump beam from the first linear LDA passes through the active element at time t_k, see Figure 4, the pump radiation intensity distribution in the volume of the active element $I_p(x_i, z_j, t_k)$ is described by the array $[I_p]$, the elements of which have the following form:

$$[I_p]_{i,j,k} = [P]_{j,k}\,[\Pi]_{i,j}, \tag{1}$$

where $[P]_{j,k} = P(z_j, t_k)$ is the pump power array element; $[\Pi]_{i,j} = \Pi(x_i, z_j)$ is the array element that determines the pump beam profile in the XZ plane in the active element:

$$\Pi\left(x_i, z_j\right) = \left(\sqrt{\frac{\pi}{2}}\, l_a \omega_p\left(z_j\right)\right)^{-1} \exp\left(-\frac{2x_i^2}{\omega_p^2\left(z_j\right)}\right). \tag{2}$$

The change in the intensity of the pump beam as it propagates along the Z axis over a distance Δz at time t_k is described by the array $[\Delta I_p]$ with elements:

$$[\Delta I_p]_{i,j,k} = -[k_p]_{i,j,k}\,[I_p]_{i,j,k}\,\Delta z, \tag{3}$$

where $[k_p]_{i,j,k} = \sigma^p_{abs}\,[N_1]_{i,j,k} - \sigma^p_{em}\,[N_2]_{i,j,k}$ is the array element of the absorption coefficient of pump radiation; σ^p_{abs} and σ^p_{em} are the absorption and stimulated emission cross sections at the pump radiation wavelength λ_p. The pump beam power in the layer with the coordinate $z_{j+1} = z_j + \Delta z$ is calculated as:

$$[P]_{j+1,k} = [P]_{j,k} + \textstyle\sum_i [\Delta I_p]_{i,j,k}\, l_a\, \Delta x, \tag{4}$$

and, accordingly, the elements of the pump beam intensity array in this layer have the form:

$$[I_p]_{i,j+1,k} = [P]_{j+1,k}\,[\Pi]_{i,j+1}. \tag{5}$$

Thus, the array of pump radiation intensities in the volume of the active element at time t_k is calculated.

After this, the elements of the population arrays of ytterbium and erbium ions energy levels are recalculated. For this, a system of rate equations, see e. g. [21], in the form of a system of finite difference equations is used:

$$\frac{[\Delta N_2]_{i,j,k}}{\Delta t} = \left[k_p\right]_{i,j,k}\left[\phi_p\right]_{i,j,k} - W_{21}[N_2]_{i,j,k} - C_{25}[N_2]_{i,j,k}[N_3]_{i,j,k}; \tag{6}$$

$$\frac{[\Delta N_3]_{i,j,k}}{\Delta t} = -C_{25}[N_2]_{i,j,k}[N_3]_{i,j,k} + \left(W_{43}^0 + C[N_4]_{i,j,k}\right)[N_4]_{i,j,k}; \tag{7}$$

$$\frac{[\Delta N_4]_{i,j,k}}{\Delta t} = W_{54}[N_5]_{i,j,k} - \left(W_{43}^0 + C[N_4]_{i,j,k}\right)[N_4]_{i,j,k}; \tag{8}$$

$$\frac{[\Delta N_5]_{i,j,k}}{\Delta t} = C_{25}[N_2]_{i,j,k}[N_3]_{i,j,k} - W_{54}[N_5]_{i,j,k}, \tag{9}$$

where h is Planck's constant; ν_p is the pump radiation frequency; $[\phi_p]_{i,j,k} = [I_p]_{i,j,k}/(h\nu_p)$.

Meaning of quantities describing the relaxation of excited levels of ytterbium and erbium ions, as well as the excitation energy transfer between them, is explained in Figure 1. Note that the probability W_{43} of the upper laser level $^4I_{13/2}$ relaxation, is understood as the total probability, i. e. the sum of the probability of relaxation W^0_{43} to the level $^4I_{15/2}$ and the probability of up-conversion relaxation $C[N_4]_{i,j,k}$ [6], see also equations (7) and (8).

The system of equations (6)–(9) is solved numerically, and the changed values of the elements of the ytterbium and erbium ion levels population arrays $[\tilde{N}_\alpha]_{i,j,k}$, ($\alpha$ = 1, 2, 3, 4, 5) due to the interaction with the pump radiation from one of the linear LDA over the time interval Δt are calculated as:

$$\begin{aligned}
\left[\tilde{N}_2\right]_{i,j,k} &= [N_2]_{i,j,k} + [\Delta N_2]_{i,j,k};\\
\left[\tilde{N}_3\right]_{i,j,k} &= [N_3]_{i,j,k} + [\Delta N_3]_{i,j,k};\\
\left[\tilde{N}_4\right]_{i,j,k} &= [N_4]_{i,j,k} + [\Delta N_4]_{i,j,k};\\
\left[\tilde{N}_5\right]_{i,j,k} &= [N_5]_{i,j,k} + [\Delta N_5]_{i,j,k}.
\end{aligned} \tag{10}$$

The distribution of the pump radiation intensity from the second linear LDA is calculated using relations (1)–(5), with the difference that the populations of the ytterbium and erbium ion levels that were established after interaction with the radiation from the first linear LDA over a time interval of Δt are used, (10). Then by using equations (6)–(9) the arrays $[\Delta\tilde{N}_\alpha]_{i,j,k}$, ($\alpha$ = 1, 2, 3, 4, 5) are calculated and the populations of the energy levels for the time moment $t_{k+1} = t_k + \Delta t$ are recalculated:

$$\begin{aligned}
[N_2]_{i,j,k+1} &= \left[\tilde{N}_2\right]_{i,j,k} + \left[\Delta\tilde{N}_2\right]_{i,j,k};\\
[N_3]_{i,j,k+1} &= \left[\tilde{N}_3\right]_{i,j,k} + \left[\Delta\tilde{N}_3\right]_{i,j,k};\\
[N_4]_{i,j,k+1} &= \left[\tilde{N}_4\right]_{i,j,k} + \left[\Delta\tilde{N}_4\right]_{i,j,k};\\
[N_5]_{i,j,k+1} &= \left[\tilde{N}_5\right]_{i,j,k} + \left[\Delta\tilde{N}_5\right]_{i,j,k}.
\end{aligned} \tag{11}$$

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At the initial moment of time, the values of the population arrays elements in the active element have the following values: $[N_1(t=0)]_{i,j} = N_{\mathrm{Yb}}$, $[N_2(t=0)]_{i,j} = 0$, $[N_3(t=0)]_{i,j} = N_{\mathrm{Er}}$, $[N_4(t=0)]_{i,j} = 0$, $[N_5(t=0)]_{i,j} = 0$, $i = 1..N_x$, $j = 1..N_z$. Here N_{Yb} and N_{Er} are the concentrations of ytterbium and erbium ions, respectively.

Elements of the upper and lower laser levels population arrays allow one to calculate the elements of the gain coefficient array of the generated radiation at the wavelength of λ_l at the time t_{k+1}:

$$\left[k_l\right]_{i,j,k+1} = \sigma^l_{em}\left[N_4\right]_{i,j,k+1} - \sigma^l_{abs}\left[N_3\right]_{i,j,k+1}. \quad (12)$$

The average-weighted gain coefficient by the volume of the generated radiation mode at time t_{k+1} is calculated as follows:

$$\bar{k}_l\left(t_{k+1}\right) = \left[k_l\right]_{k+1} = \frac{\Delta x \Delta z l_a}{V_l}\sum_{i,j}\left[k_l\right]_{i,j,k+1}\left[\Lambda\right]_{i,j}, \quad (13)$$

where $[\Lambda]_{i,j} = \exp(-2(x^2_i + z^2_j)/\omega^2_l)$ is the Gaussian function that describes the fundamental mode of the generated radiation; $V_l = (\pi\omega^2_l\, l_a)/2$ is the effective volume of the generated mode.

This algorithm allows one to calculate the average-weighted gain coefficient as a function of time, $\bar{k}_l(t_k) = [k_l]_k$, during the entire pump pulse. The gain coefficient corresponding to the end of the pump pulse will be denoted as $k_l^{\max}$.

It should be noted that the proposed algorithm for calculation the gain in the active element allows us to determine values of parameters of the active medium, resonator and pumping system at which the generation threshold is reached, which correlate well with the experimentally obtained results, see below section 3 of the article.

2.3. Equations for calculating the output characteristics of a laser

To start the formation of a single pulse in solid-state lasers with passive Q-switching, the gain coefficient must be equal to the initial coefficient of total losses in the resonator (the condition for reaching the "first threshold") [8]. In [19], it was shown that this condition can be written as $\bar{k}_l(t) = \sigma_l N_l^i$, where $\sigma_l = \sigma^l_{abs} + \sigma^l_{em}$, and the parameter N_l^i determines the volumetric average population of the upper laser level at the initial moment of formation of the laser pulse:

$$N_l^i = \frac{k_L l_a - \ln\left(T_0\right)}{\sigma_l l_a}. \quad (14)$$

Here $k_L = -[\ln(1-T_{out}) + \ln(1-L)]/(2l_a)$ is the resonator total loss coefficient; T_{out} is the output mirror transmission coefficient; L is the passive resonator loss; T_0 is the initial transmission of the saturable absorber.

In our case, the laser operates in the single pulse regime, and therefore it is sufficient that the following condition is satisfied by the end of the pump pulse to reach the "first threshold":

$$k^l_{\max}/\sigma_l > N_l^i. \quad (15)$$

Parameters of the single pulse generated by the laser, according to [19], can be calculated as follows. Laser output pulse energy:

$$E_{out} = V_l k_{act} h\nu_l \frac{1}{\sigma_l}\ln\left(N_l^i / N_l^f\right), \quad (16)$$

where $k_{act} = -\ln(1-T_{out})/(2l_a)$ is the coefficient of active loss of the resonator.

Peak power of the light pulse at the laser output is:

$$P_{out}^{peak} = V_l k_{act}\Phi_l\left(N_l^t\right). \quad (17)$$

The laser pulse duration τ_l is calculated using the formula:

$$\tau_l = E_{out} / P_{out}^{peak}. \quad (18)$$

The parameters N_l^t, N_l^f and $\Phi_l\,(N_l^t)$ required to calculate the laser output characteristics from (16), (17) and (18), can be found from equations (19), (20), (21) [19]:

$$\frac{N_l^t}{N_l^i} - \frac{N_l^0}{N_l^i} - \left(1 - \frac{N_l^0}{N_l^i}\right)\left(\frac{N_l^t}{N_l^i}\right)^{\alpha} = 0; \quad (19)$$

$$1 - \frac{N_l^f}{N_l^i} + \frac{N_l^0}{N_l^i}\ln\left(\frac{N_l^f}{N_l^i}\right) - \frac{1}{\alpha}\left(1 - \frac{N_l^0}{N_l^i}\right)\left[1 - \left(\frac{N_l^f}{N_l^i}\right)^{\alpha}\right] = 0; \quad (20)$$

$$\Phi_l\left(N_l^t\right) = \frac{c\mu}{n}\left[\begin{array}{l} N_l^i - N_l^t - \dfrac{k_L l_a + \beta\ln\left(\dfrac{1}{T_0}\right)}{l_a\sigma_l}\ln\left(\dfrac{N_l^i}{N_l^t}\right) - \\ -\dfrac{(1-\beta)\ln\left(\dfrac{1}{T_0}\right)}{l_a\sigma_l\alpha}\left(1 - \left(\dfrac{N_l^t}{N_l^i}\right)^{\alpha}\right)\end{array}\right], \quad (21)$$

where $\alpha = (\xi\sigma_{gs})/\sigma_l$; $\beta = \sigma_{es}/\sigma_{gs}$; $N_l^0 = (k_L l_a + \beta\ln(1/T_0))/(l_a\sigma_l)$; $\Phi_l(N_l^t)$ is the peak lasing photon flux N_l^t and N_l^f are the effective populations of the upper laser level corresponding to the peak value of the generated photon flux and the end of the generation pulse, respectively; σ_{gs}, σ_{es} are the absorption cross-sections from the ground and excited states of the saturable absorber, respectively; $\xi = A_l / A_{sa}$ is the ratio of the effective area of the generation mode in the active medium and the passive modulator; c is the speed of light in vacuum; n is the refractive index of the active medium; $\mu = (nl_a)/l_{cav}$ is the filling factor of the resonator; l_{cav} is the optical length of the resonator.

3. Calculation of generation characteristics of Yb, Er phosphate glass lasers in passive Q-switch regime. Comparison with experimental data

Mathematical model of the laser proposed in this work was used in the development of passive Q-switched lasers with a phosphate glass active elements with ytterbium and erbium ions using a thin plate of Co^{2+}:MALO as saturable absorber with transverse pumping by radiation from linear LDAs.

Geometrical dimensions of active elements are as follows length $l_a = 10$ mm, thickness $t_a = 1$ mm and height $h_a = 2$ mm. The main spectroscopic parameters of ytterbium and erbium ions in the active medium are given in Table 1.

Wavelengths of pump λ_p and laser λ_l radiation are 940 nm and 1540 nm, respectively. The absorption cross section from the ground σ_{gs} and excited σ_{es} states of the saturable absorber Co^{2+}:MALO is $\approx 4.0 \cdot 10^{-23}$ m^{-2} and $>3.0 \cdot 10^{-24}$ m^{-2}, respectively [8]. Thickness of the passive modulator is 0.5 mm. As can be seen from the presented data, $\sigma_{es} << \sigma_{gs}$ and $\sigma_{gs} > \sigma^l_{em} + \sigma^l_{abs}$, which means that the "second threshold" is fulfilled, when the absorption of the passive modulator is saturated firstly, and then the gain of the active medium is. This ensures the generation of a laser pulse [8].

Table 1

Spectroscopic parameters of ytterbium and erbium ions in phosphate glass

Yb^{3+}(*)			Er^{3+} [6]					
W_{21}, s^{-1}	σ^p_{abs}, m^{-2}	σ^p_{em}, m^{-2}	W^0_{43}, s^{-1}	C, m^3/s	W_{54}, s^{-1}	C_{25}, m^3/s	σ^l_{abs}, m^{-2}	σ^l_{em}, m^{-2}
833	0.27 10^{-24}	0.5 10^{-25}	125	10^{-24}	$5 \cdot 10^5$	4.1 10^{-22}	$8 \cdot 10^{-25}$	$8 \cdot 10^{-25}$

(*) our measurements.

For ease of comparison of calculated results with experimental data, the radiation power P_p of the linear LDA is presented as a function of the current I_{LD} feeding the LDA:

$$P_p = KI_{LD} + B,$$

where $K = 1.25$ W/A and B = -25 W. Current I_{LD} is measured in amperes.

Calculations by the ABCD method show that the Gaussian radius of the generated beam in the active element changes by less than 1 %. The saturable absorber was located near the active element. Therefore, this radius was constant along the length of the active element and the saturable absorber.

Figure 5 shows results of laser operation modeling with erbium and ytterbium ion concentrations in the active element $N_{Er} = 0.5 \cdot 10^{26}$ m^{-3} and $N_{Yb} = 2.1 \cdot 10^{27}$ m^{-3} (element "A"). The initial transmission of the passive modulator $T_0 = 95.5$ %, the geometric length of the resonator is 90 mm, the distance from the linear LDAs to the side surfaces of the active element is 0.6 mm, the pump pulse duration is 2 ms, the Gaussian radius of the generated radiation beam is $\omega_l = 380$ µm.

Table 2 presents the calculated and experimental values of E_{out} and τ_l of the passive Q-switched laser based on active element "A" at different values of the output mirror transmission and the current of the linear LDAs.

Table 2

Calculated and experimental characteristics of the laser based on the active element "A". The experimental values of the parameters are given in brackets

T_{out}, %	I_{LD}, A	E_{out}, mJ	τ_l, ns
10	60	1.0 (1.1)	40.9 (43.4)
15	70	1.1 (1.2)	41.0 (43.3)
20	80	1.2 (1.2)	42.3 (42.6)
27.5	85	1.2 (1.4)	45.7 (41.5)

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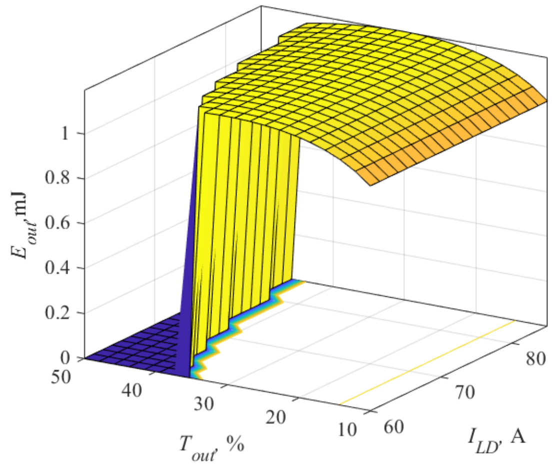
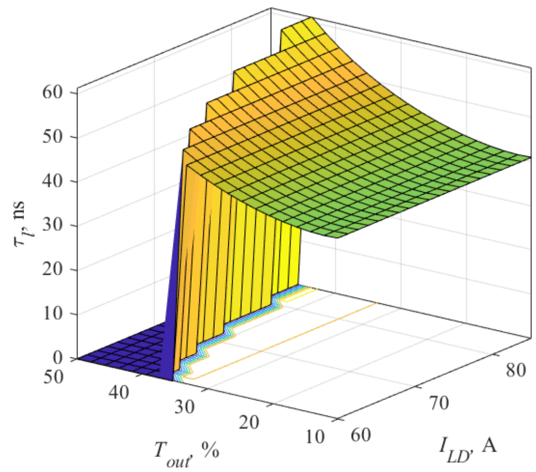


a



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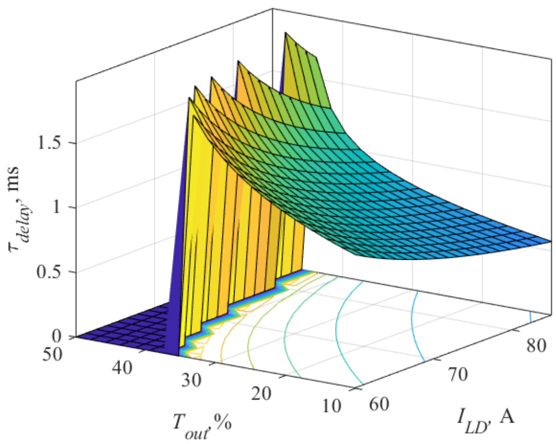

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Figure 5 – Calculated characteristics of a laser based on the active element "A" depending on I_{LD} and T_{out}: *a*, *b* – energy E_{out}, and duration τ_l of the output pulse, respectively; *c* – time delay between the start of the pump pulse and the start of the generation pulse τ_{delay}

Calculations of lasers' parameters based on active element with ytterbium and erbium ion concentrations $N_{Er} = 0.7 \cdot 10^{26}$ m^{-3} and $N_{Yb} = 3.5 \cdot 10^{27}$ m^{-3} (element "B") with initial transmissions of passive modulators $T_0 = 95.5$ % and 91.5 % were also performed. The distance from the linear LDAs to the side surface of the active element is 0.7 mm, the geometric length of the resonator is 90 mm. The Gaussian radii of the generated radiation beams are given in Table 3, the pump pulse duration is 2.5 ms.

Table 3

Output characteristics of lasers with the element "B", at $T_0 = 95.5$ % and 91.5 %. Experimental data are given in brackets

T_0, %	ω_l, µm	T_{out}, %	I_{LD}, A	E_{out}, mJ	τ_l, ns
95.5	380	20	57.5	1.16 (1.05)	42.3 (43.5)
		27.5	62.5	1.19 (1.15)	45.7 (42.5)
91.5	360	20	67.5	1.9 (2.1)	21.2 (25)
		27.5	72.5	2.0 (2.2)	21.2 (23)

Figures 6 and 7 show results of lasers' modeling with the active element "B" and passive modulators with $T_0 = 95.5$ % and 91.5 %.

Table 3 shows the results of calculations of the parameters of lasers with the active element "B" for some values of the initial transmission of the passive modulator, the transmission of the output mirror and the current of the linear LDAs in comparison with experimentally obtained data.

The graphical dependencies of the laser characteristics presented in Figure 5–7 allow one to determine the range of T_{out} and I_{LD} values at which the generation threshold is reached at a given value of T_0. Also, from these graphs, one can determine the values of T_{out} for obtaining the maximum energy of the generated pulse or its minimum duration.

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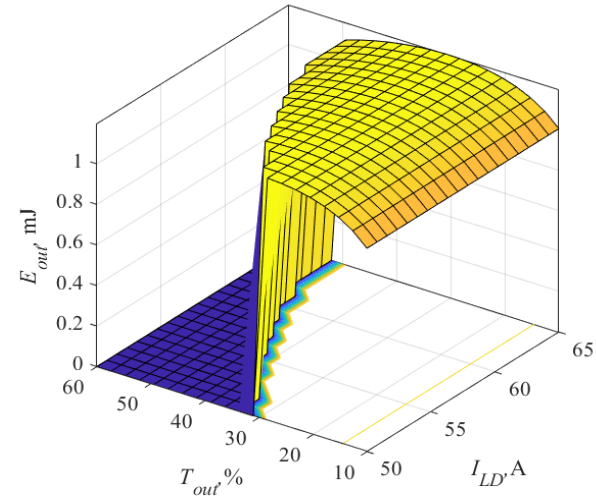

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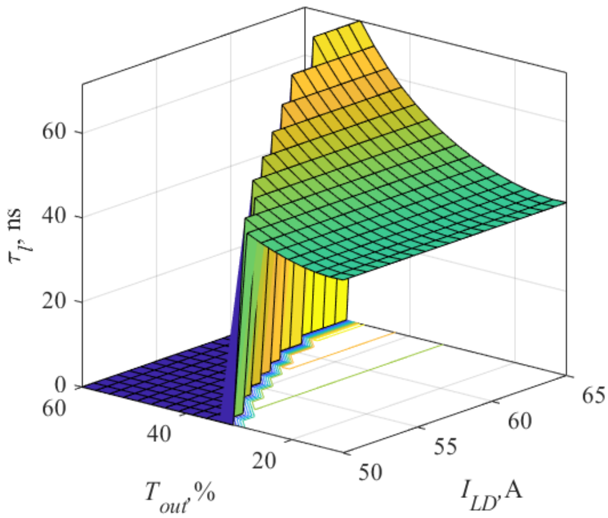

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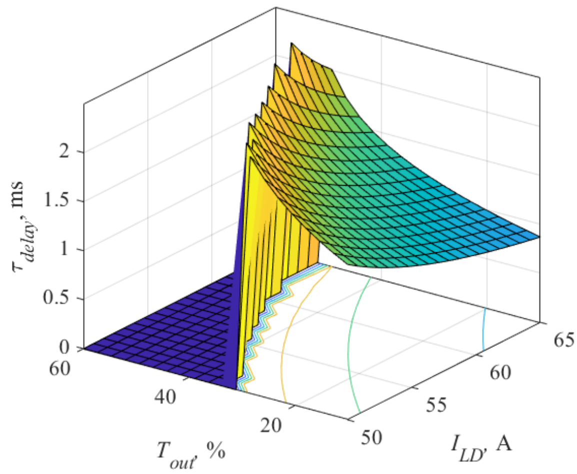

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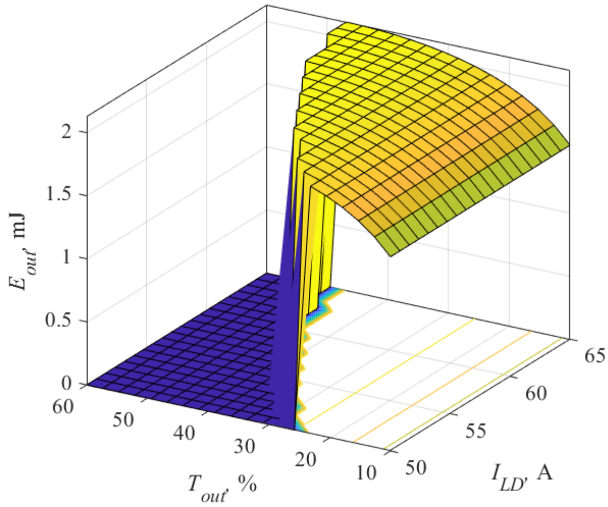
Figure 6 – Output characteristics of the laser based on the active element "B", T_0 = 95.5 %, depending on I_{LD} and T_{out}: *a*, *b* – energy E_{out}, and duration τ_l of the output pulse, respectively; *c* – time delay between the start of the pump pulse and the start of the generation pulse τ_{delay}



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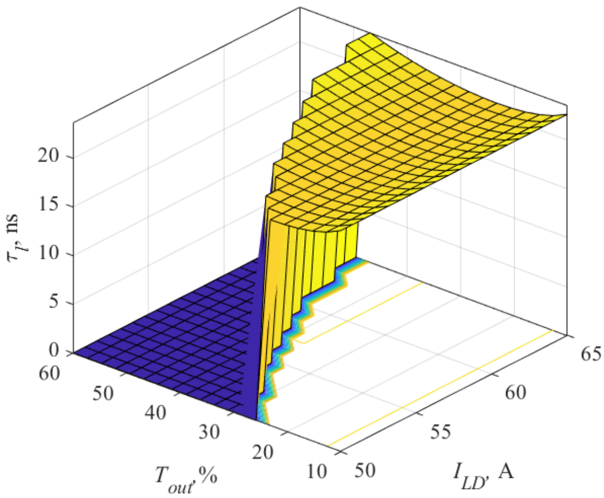

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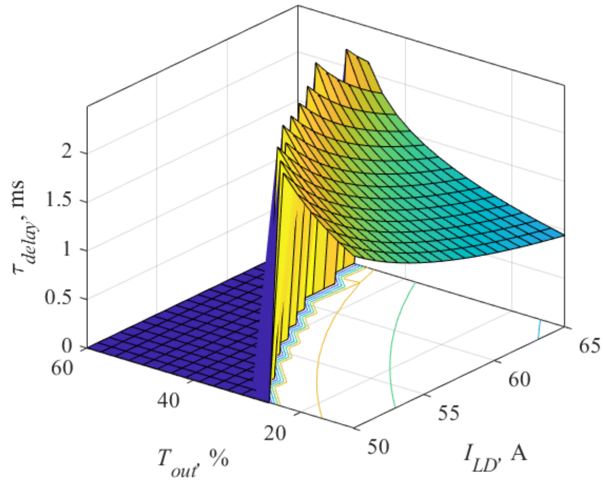

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Figure 7 – Output characteristics of the laser based on the active element "B", T_0 = 91.5 %, depending on I_{LD} and T_{out}: *a*, *b* – energy E_{out}, and duration τ_l of the output pulse, respectively; *c* – time delay between the start of the pump pulse and the start of the generation pulse τ_{delay}

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Analysis of results presented in Tables 2 and 3 shows that the calculated values of the lasers' output characteristics under study are in good quantitative agreement with the experimental data.

Conclusion

A mathematical model of a passively Q-switched solid-state laser based on ytterbium and erbium co-doped active media with transverse pumping by linear laser diode arrays has been developed. The gain in the laser is calculated using rate equations taking into account the space-time dependence of the pump radiation intensity in the laser element. Laser output characteristics are calculated using analytical equations obtained for a passively Q-switched laser in the approximation of a "slow" saturable absorber. The model allows one to find the range of parameters of the active element, passive modulator, resonator, and pumping system at which the generation threshold is reached and the laser generates pulses with the required energy and duration.

Modeling results were used for experimental development of lasers with active element based on phosphate glass doped by ytterbium and erbium ions. The energy and duration of the output light pulses were ≈1 mJ, ≈40 ns, and ≈2 mJ and ≈20 ns, depending on the content of ytterbium and erbium ions in the active element, as well as on the initial transmission of the passive modulator and the resonator parameters.

Within the framework of the developed model parameters of a laser with several active elements with different concentrations of ytterbium and erbium ions as well as with different parameters of the resonator and LDAs can be optimized to obtain generated pulses with the required energy and duration.

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