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Experimental Verification of Laser Radar Cross Section Model for Complex Targets with Gaussian Beam Irradiation

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Abstract

The reflection of a Gaussian laser beam from a flat Lambert disk is considered theoretically. It was found that the results of experimental measurements of the reflected beam power as a function of the disk radius at various distances from the photodetector to the target are in good agreement with the theoretical model. It was shown that when the radius of the laser beam is greater than the dimensions of the probed complex target this target can be replaced by an equivalent Lambert disk with the same laser radar cross section.

Keywords: laser scanning, laser radar cross section, lidar, gaussian beam

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Экспериментальная проверка модели эффективной площади отражения сложных объектов в гауссовом лазерном пучке

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Теоретически рассмотрено отражение лазерного гауссова пучка от плоского круглого ламбертова диска. Установлено, что результаты экспериментальных измерений мощности отражённого пучка в зависимости от радиуса диска при различных дистанциях от фотоприёмной системы до объекта хорошо согласуются с теоретической моделью. Показано, что при радиусе лазерного пучка больше размеров зондируемого объекта сложной формы, его можно заменить эквивалентным ламбертовым диском с такой же эффективной площадью отражения.

Ключевые слова: лазерное сканирование, эффективная площадь отражения, лидар, гауссов пучок

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Introduction

Application of LiDARs for detecting, recognizing, and tracking objects has recently expanded significantly. The most important characteristic for estimation of the target detection range is its laser radar cross section (LRCS) [1–4]. It depends on the geometric shape, reflective properties, and orientation of the object. Traditionally, LRCS is measured by comparison with the standard one [5, 6]. At the same time, numerical modeling methods for LRCS estimation have been significantly developed [6–10]. The theoretical calculations of the LRCS are known for some objects with simple geometric shapes [2, 7]. This is especially important because an object can be represented as a set of elements that are in most cases simple in shape and thus one obtains a quick LRCS estimation of a complex object. However, numerical modeling requires knowledge of many object parameters, such as the bidirectional reflection distribution function [11, 12], its exact geometric shape, and from what materials the object is made of. In theoretical analysis many significant simplifications have to be applied. So an object is usually considered as to be illuminated with a laser beam having a uniform power distribution, whereas in reality the beam has often a Gaussian distribution in space. In addition – simulation results are often difficult to verify experimentally.

In this paper, we for the first time (as far as we know) theoretically consider the reflection of a Gaussian beam from a circular disk, which reflects according to Lambert's law. Experimental measurements of a reflected laser beam power are given as a function of the disks' radii, the distance to them, and for a model of an object with complex geometric shape. A method for estimating the reflected radiation power simplified in comparison with the radar equation based on the construction of an equivalent Lambert disk with an LRCS equal to the target LRCS one is proposed.

Theoretical analysis

Let us assume that the radiation source generates a laser pulse with a given temporal and spatial distribution of the radiation power density $I'(\boldsymbol{r}, t)$. Laser radiation emitted by a small section of the source $d\sigma_s$ illuminates a small area of the object dS with the normal vector **n** located in the direction $\boldsymbol{r}_1$ from $d\sigma_s$ after passing through the propagation medium with the transmission coefficient T_1 (Figure 1). After interacting with the object the radiation passes through the propagation medium with the transmission coefficient T_2 in the direction $\boldsymbol{r}_2$ and enters the receiver small section $d\sigma_d$ with the registration efficiency η. The fraction of radiation $\rho(\boldsymbol{r}_1, \boldsymbol{r}_2)$ (bidirectional reflection distribution function) reflected towards the receiver element $d\sigma_d$ depends on the optical properties of the object surface element dS, its relative orientation determined by angles between the normal vector and the directions to the sections of the source $d\sigma_s$ and the receiver $d\sigma_d$.

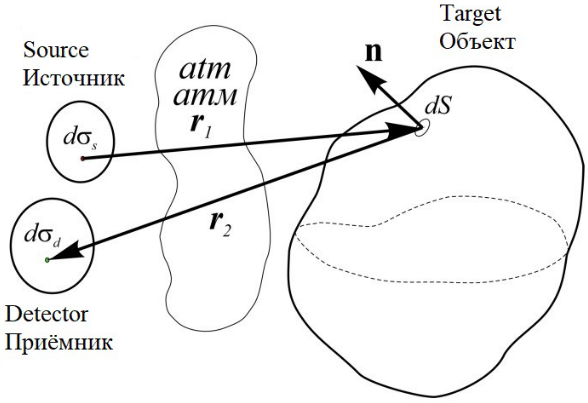

Figure 1 – Scheme of object probing by a laser pulse

The power $d^3P(t)$ received by the photodetector element at time t is equal to [4]:

$$d^3P(t) = I'\left(t'\right)T_1T_2\frac{\rho(\boldsymbol{r}_1,\boldsymbol{r}_2)}{r_1^2r_2^2}\frac{(-\boldsymbol{r}_1\cdot\boldsymbol{n})}{r_1}\frac{(\boldsymbol{r}_2\cdot\boldsymbol{n})}{r_2}\eta dSd\sigma_s d\sigma_d, \quad (1)$$

where $t' = t - l(\boldsymbol{r}_1, \boldsymbol{r}_2)/c$, $l(\boldsymbol{r}_1, \boldsymbol{r}_2)$ is the optical path length in the forward and backward directions; c is the speed of light.

Denote the total power falling from the source to the object as P_a:

$$P_a(t) = \iint_{\sigma_s S}\frac{I'}{r_1^2}dSd\sigma_s = 2\pi r^2 I_{av}(t), \quad (2)$$

where the integration is carried out over a part of the object surface S the radiation reflected from which will arrive at the input of the receiving system at the same time, I_{av} is the average power density of the laser beam, r is the distance from source to target. The reflected from the object power will be proportional to LRCS which based on Eq.(1) and with taking into account Eq.(2) is equal to:

$$\sigma(t) = \frac{1}{P_a}\iint_{\sigma_s S}I'\rho(\boldsymbol{r}_1,\boldsymbol{r}_2)\frac{(-\boldsymbol{r}_1\cdot\boldsymbol{n})}{r_1}\cdot\frac{(\boldsymbol{r}_2\cdot\boldsymbol{n})}{r_2}\cdot dS\cdot d\sigma_s.$$

If the distance to the probed objects is much greater than the distance between the source and the receiver and also greater than their sizes and the object size then $r_1 \approx r_2 = r$ and therefore $T_1 \approx T_2 = T$ can be taken. The following formula is obtained from Eq.(1):

$$P(t) = T^2 \frac{I_{av}(t - 2r/c)}{2\pi r^2} \sigma\left(t - \frac{2r}{c}\right)\sigma_d \eta, \qquad (3)$$

where σ_d is the area of the entrance pupil of the receiver objective, and it is assumed that the sensitivity of the photodetector is the same over the entire surface of it. Thus LRCS can be defined as the area of a certain flat surface, which is located perpendicular to the direction of the incident wave and reflects the same power in the direction of the photodetector as the probed object when it's placed at the point of location of the that object. In general, LRCS is a function of time because different parts of the object's surface are illuminated at different times during probing by a laser pulse.

We consider the reflection of laser radiation from a Lambert circular disk with $\rho(\boldsymbol{r}_1, \boldsymbol{r}_2) = \rho_t = \text{const}$ of radius r_t located at a distance z from the receiver and oriented perpendicular to the axis of the laser beam. Let the radiation source forms a Gaussian beam in the plane of the probed disk with the power density in the cross section:

$$I(r') = I_0 \cdot e^{-\frac{r'^2}{w^2}},$$

where $I_0 = P_0/(\pi w^2)$ is the radiation power density at the center of the beam; P_0 – laser power; w is the radius of beam where irradiance decreases by a factor of $1/e$. The reflected power of the radiation arriving at the input aperture of the receiver objective is:

$$P = \int_{\sigma_d}\int_S \frac{I_0 \cdot e^{-r'^2/w^2}}{2\pi r_{td}^2} \rho_t \eta \cdot dS d\sigma_d,$$

where r_{td} is the distance from the small disk element dS to the receiver element $d\sigma_d$ (Figure 2); $dS = r'dr'd\varphi'$, and r' ranges from 0 to r_t.

In the approximation $r_{td}^2 = z^2 + r'^2$ for $r_{td} >> r_d$ (r_d is the radius of the aperture of the receiver objective), it can be assumed that a weakly varying in power amplitude electromagnetic field is created in the plane of the photodetector. If we assume that the sensitivity of the receiving area is uniform ($\eta = \text{const}$), then the integral can be transformed to the equation:

$$P = I_0 \frac{\rho_t \eta \sigma_d}{2\pi} \int_0^{r_t} \frac{e^{-r'^2/w^2}}{z^2 + r'^2} 2\pi r' dr' =$$

$$= \frac{1}{2} I_0 \rho_t \eta \sigma_d \cdot e^{\frac{z^2}{w^2}} \int_{z^2/w^2}^{(r_t^2+z^2)/w^2} \frac{e^{-x}}{x} dx.$$

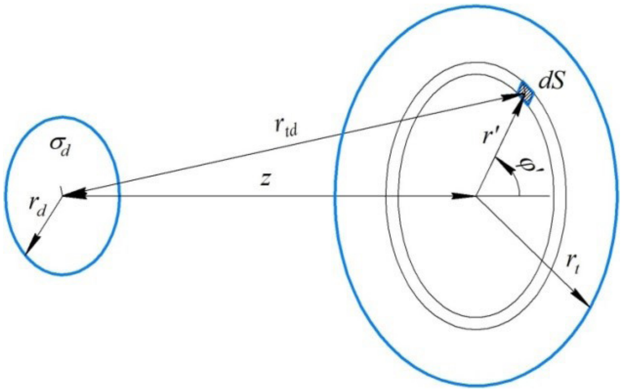

Figure 2 – Scheme for calculating the power of reflected radiation

Taking into account the definition of the integral exponential function of the first kind [13]:

$$\text{Ei}_1(x) = \int_x^\infty \frac{e^{-t}}{t} dt,$$

we obtain:

$$P = \frac{1}{2} I_0 \rho_t \eta \sigma_d \cdot e^{\frac{z^2}{w^2}} \left[\text{Ei}_1\left(\frac{z^2}{w^2}\right) - \text{Ei}_1\left(\frac{r_t^2 + z^2}{w^2}\right)\right].$$

In most cases, this expression can be simplified by considering that $z >> r_t$ and $z >> w$:

$$P = \frac{I_0 \rho_t \eta \sigma_d w^2}{2z^2}\left[1 - e^{-\frac{r_t^2}{w^2}}\right]. \qquad (4)$$

From this equation the LRCS of the Lambert disk is equal to:

$$\sigma_t = \pi w^2 \rho_t \left[1 - e^{-\frac{r_t^2}{w^2}}\right]. \qquad (5)$$

This is consistent with the results [1, 8]. In this case the radius of the Gaussian beam changes with distance according to the expression:

$$w^2 = w_0^2\left[1 + \left(\frac{\lambda z'}{\pi w_0^2}\right)^2\right], \qquad (6)$$

where w_0 is the beam waist radius; $z' = z + z_0$; z_0 is the distance from the Gaussian beam waist to laser lens; λ is the radiation wavelength. Hence the LRCS also depends on the distance z.

The obtained expressions are valid if the center of the laser Gaussian beam coincides with the center of the probed disk. If the center of the disk is displaced from the beam axis, the expressions become much more complicated. The reflected power in this case can be found using the Eq.(A.2) (Appendix A).

Experimental results and discussion

From the Eq.(5) it follows that if the beam width is much less than the target size $w << r_t$ then LRCS increases with distance z according to the quadratic law $\sigma_t = \rho_t \pi w^2$ where w is proportional to z in accordance with Eq.(6). Since irradiance at the center of the beam I_0 decreases in proportion to z^{-2} in Eq.(4), the power P recorded by the photodetector will also decrease in proportion to z^{-2}. However, as the ratio r_t/w decreases, the increase of LRCS slows down, and it itself rapidly tends to the value $\rho_t \pi r_t^2$. As a result, at $w \geq r_t$ the measured power starts to decrease faster in proportion to z^{-4}. Therefore, it should seek for scanning by a laser beam with a small radius to detect small objects.

At the same time wide beam scanning has its advantages. Full coverage of the target by the beam allows LRCS to be obtained from the entire object and therefore the measured signal will contain information about the entire object. Measurement of the time dependence of a reflected power from the target allows it to be used for object recognition [2]. Whereas in the case of a narrow beam only a part of the object is measured and it is difficult to ensure that two successive laser pulses hit the same part of the object. Therefore the time dependence of the LRCS obtained with a narrow laser beam is not suitable for obtaining additional information about an object and determining its type.

To test the theoretical model (4), the power of the Gaussian beam reflected from the objects was measured experimentally. All elements of the experimental setup for measuring the power of the reflected laser beam are located in the non-reflective box to reduce the influence of external factors (Figure 3). The source of laser radiation was a DPSS laser manufactured by "Thorlabs, Inc." CPS532 with a wavelength of 532 nm. To form the required radiation divergence (0.026 rad), an objective with a focal length of -6.25 cm was developed. To measure the LRCS of complex-shaped objects, they were placed on a rotating and moving device that sets their different orientations. After reflection from the object, the radiation is received by a Gentec-EO PH100-Si-HA-OD1-D0 photodetector located at the same distance from the object as the exit pupil of the laser source objective. Data from the photodetector are converted by the Gentec-EO U-LINK (USB) PC interface and output to the PC as radiation power values. In this case, the distance between the source and the receiver of radiation is much less than the distance to the object.

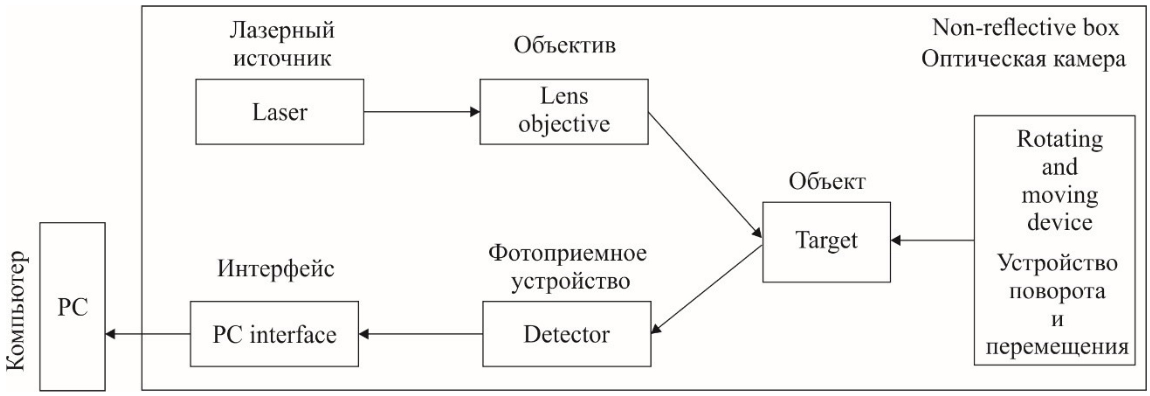

Figure 3 – Scheme of the experimental setup

To determine the parameters of the laser beam formed by the lens, the photodetector was placed on the rotating and moving device. The beam profile was measured at several distances from the laser radiation source. Detailed measurements of the laser beam profile showed that it has some differences from the Gaussian profile. For the laser we used, the maximum deviations do not exceed 10 %, and the average deviation is less than 4 %, which allows us to use the Gaussian beam model. The laser beam radius was defined as the radius of the circle on which the power density drops by a factor of e. The change in the laser beam radius depending on the distance is in complete agreement with expression (6) for a waist width of $w_0 = 6.3$ µm and its position relative to the laser exit pupil of -17.7 cm.

Paper flat discs were chosen as targets. The radius of the disks r_t varied in the range of 1.5–20 cm. The choice of paper as a material for discs is due to the fact that it reflects almost according to Lambert's law, and the reflection coefficient is relatively high: $\rho_t = 0.67$. The beam radius was 8–11 cm at distances from 300 to 400 cm. The results of measurements of the reflected power at four distances z from the source to the target are shown in Figure 4. For all four distances, the experimental values are well described by dependencies (4) with $P_0 = 4.1$ mW, $\eta\sigma_d = 0.9$ cm^2.

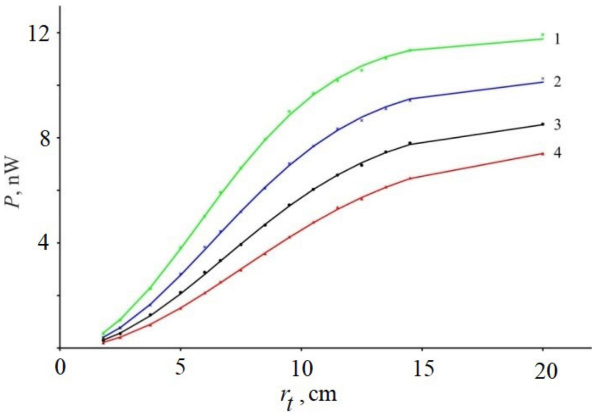

Figure 4 – Radiation power P reflected from a target with radius r_t located at a distance of 315 (1), 340 (2), 370 (3), and 400 cm (4): dots stand for the experimental values, lines stand for the theoretical curves of the Eq.(4)

Measurements of the reflected power from disks made of different materials with different reflection coefficients confirm theoretical results for distances from 250 to 400 cm. The results for the discs placed at a distance 315 cm are shown in Figure 5. The reflection coefficients are 0.64 (plastic), 0.67 (paper), 0.515 (textile), 0.45 (wood).

Let's assume that the LRCS of an object with a complex geometric shape is known. Then you can associate it with a flat disk with a diameter approximately equal to the transverse dimensions of the object and can calculate the reflection coefficient from the Eq. (5). The diameter should be chosen based on the size of the object as it is important to keep the r_t/w ratio approximately the same as for the real target. Further a simpler Eq. (4) can be used instead of the Eq. (3) which requires knowledge of the dependence of LRCS on the distance to the object.

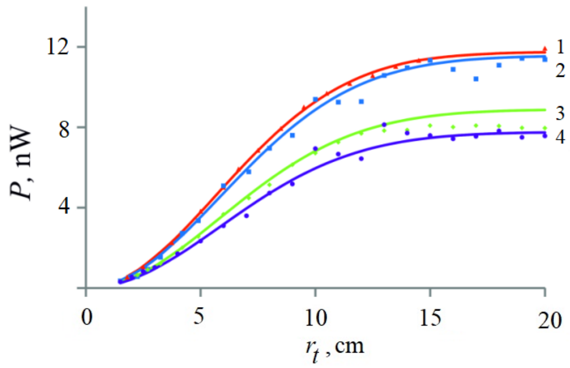

Figure 5 – Radiation power P received by the photodetector, depending on the material of disks: paper (1), plastic (2), textile (3), wood (4), the solid line is stand for the theoretical curve of the Eq.(4)

Measurements were carried out for various objects with complex surface shapes. Figure 6 shows the results of comparison of received power after reflection from a T-34 tank model (scale 1:43) and from an equivalent disk with a radius of 5 cm. The reflection coefficient of the disk was 0.25.

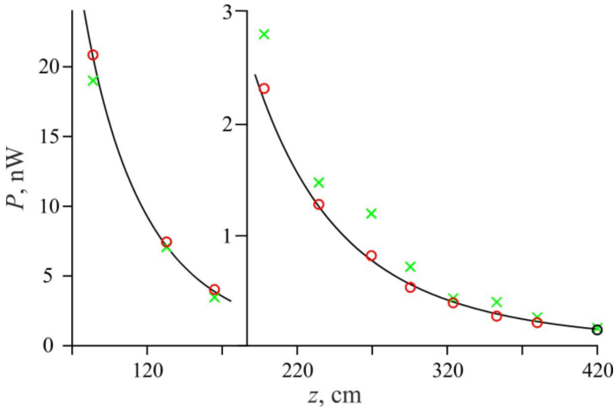

Figure 6 – Radiation power P received by the photodetector, depending on the distance z to the target: crosses stand for the measured reflected power from the T-34 tank model, circles stand for the disk with laser radar cross section equal to the laser radar cross section of the model, the solid line is stand for the theoretical curve of the Eq.(4)

Similar dependencies were obtained for other objects. As expected the largest discrepancies are observed at small distances where the beam width is less than the target dimensions. As the distance to the target increases the beam radius increases and the values agree much better. Thus replacing a complex object with an equivalent LRCS disk allows to obtain correct estimations of the reflected power at large distances to the target. Measurements of the power of the reflected signal at several distances to the object make possible to estimate the size of the probed object. Good quantitative agreement of the theoretical equation (4) with experimental measurement results allows it to be used for estimate the maximum detection range of a target. The maximum range can be expressed from equations (4)–(6) provided that the power received by the receiver is the minimum registered power $P_{\min}$, at which it is still possible to detect an object against the background of noise, and provided that $z >> w_0 r_t/\lambda$ i. e. when the LRCS of the object tends to the value $\rho_t \pi r_t^2$. Thus, the maximum range is:

$$z_{max}^4 = \frac{\pi P_0 w_0^2 \eta \sigma_d \rho_t r_t^2}{2\lambda^2 P_{min}}.$$

In our case, at $P_{\min} = 0.15$ nW, the maximum detection range of the T-34 tank model was 4.3 m.

Conclusions

The paper presents a theoretical model of a Gaussian beam power reflected from a Lambert circular reflector. Dependence of the laser radar cross section on the distance between the radiation source and the target is considered. Powers of reflected laser radiation from disks of different radii are measured experimentally. Theoretical calculations are in good agreement with experimental data. Measurements of the reflected from a complex-shaped object radiation power have shown that dependence of the reflected light power on the distance to the object agrees well with the such dependence for disks. The model can be used for calculation the maximum detection range to an object with a known laser radar cross section as well as for estimation of the power registered by the photodetector.

Appendix A. Beam center offset correction

Let us consider how the displacement of the center of the disk from the center of the laser spot will affect the power received by the photodetector. Let's choose the center of the disk as the origin. Let the center of the beam be displaced along the *OX* axis by a small distance $r_0 << w$. Then the power density can be written as:

$$I(r') = I_0 e^{-\frac{(r'\cos\varphi - r_0)^2 + (r'\sin\varphi)^2}{w^2}},$$

where r' and φ – are polar coordinates. It is easy to convert this expression to the form:

$$I(r') = I_0 e^{-\frac{r_0^2}{w^2}} \cdot e^{-\frac{r'^2}{w^2}} \cdot e^{\frac{2r' r_0 \cos\varphi}{w^2}}.$$

Further we will assume that the distance to the disk z is much greater than the beam radius and the disk radius r_t. Then the power reflected from the disk and is incident onto the entrance pupil of the receiving optical system, is equal to:

$$P = I_0 \frac{\rho_t \eta \sigma_d}{\pi z^2} e^{-\frac{r_0^2}{w^2}} \int_0^{2\pi}\int_0^{r_t} e^{-\frac{r'^2}{w^2} \frac{2r' r_0 \cos\varphi}{w^2}} r' dr' d\varphi.$$

To calculate the integral we note that the second exponential component has a small value in the integral and so it can be expanded into a series:

$$e^{\frac{2r' r_0 \cos\varphi}{w^2}} \approx 1 + \frac{2r' r_0 \cos\varphi}{w^2} + \frac{1}{2}\left(\frac{2r' r_0 \cos\varphi}{w^2}\right)^2 + \ldots$$

Then:

$$P \approx I_0 \frac{\rho_t \eta \sigma_d}{\pi z^2} e^{-\frac{r_0^2}{w^2}} \cdot \left[\int_0^{2\pi}\int_0^{r_t} e^{-\frac{r'^2}{w^2}} r' dr' d\varphi + \int_0^{2\pi}\int_0^{r_t} e^{-\frac{r'^2}{w^2}} \frac{2r' r_0 \cos\varphi}{w^2} r' dr' d\varphi + \frac{1}{2}\int_0^{2\pi}\int_0^{r_t} e^{-\frac{r'^2}{w^2}} \left(\frac{2r' r_0 \cos\varphi}{w^2}\right)^2 r' dr' d\varphi \right].$$

The first term in the expression is:

$$\int_0^{2\pi}\int_0^{r_t} e^{-\frac{r'^2}{w^2}} r' dr' d\varphi = \pi w^2 \left[1 - e^{-\frac{r_t^2}{w^2}}\right].$$

The second term is zero and the third is:

$$\frac{1}{2}\int_0^{2\pi}\int_0^{r_t} e^{-\frac{r'^2}{w^2}} \left(\frac{2r' r_0 \cos\varphi}{w^2}\right)^2 r' dr' d\varphi = \pi r_0^2 \left[1 - \left(1 - \frac{r_t^2}{w^2}\right) \cdot e^{-\frac{r_t^2}{w^2}}\right].$$

After all the simplifications we obtain the final expression for the power received by the photodetector:

$$P = I_0 \frac{\rho_t \eta \sigma_d}{\pi z^2} \cdot e^{-\frac{r_0^2}{w^2}} \cdot \pi w^2 \times \left[\left(1 - \frac{r_0^2}{w^2}\right)\left(1 - e^{-\frac{r_t^2}{w^2}}\right) + \frac{r_0^2 r_t^2}{w^4} \cdot e^{-\frac{r_t^2}{w^2}}\right]. \quad \text{(A1)}$$

To calculate the relative error we find the ratio of the power difference P_1 according to the Eq.(4) and P_2 according to the Eq.(A.1) to P_1:

$$\varepsilon = \frac{P_1 - P_2}{P_1} = 1 - \left(1 - \frac{r_0^2}{w^2}\right) \cdot e^{-\frac{r_0^2}{w^2}} - \frac{r_0^2 r_t^2}{w^4} \cdot \frac{1}{e^{r_t^2/w^2} - 1}. \qquad \text{(A2)}$$

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