Non-Additive Quantity Measurement Model

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Abstract

This work considers a model for measuring non-additive quantities, in particular a model for subjective measurement. The purpose of this work was to develop the measurement theory and form of a measurement model that uses the corrected S. Stevens measurement model.

A generalized structure was considered that included an empirical system, a mathematical system, and a homomorphism of the empirical system into a numerical system. The main shortcomings of classical measurement theories seem to be: 1) homomorphism does not display operations (in this case, one cannot speak of the meaningfulness of the model); and 2) there is no empirical measurement model that could confirm the existence of a homomorphism. To overcome the shortcomings of existing theories a definition of the measurement equation is given. As a result a measurement model is obtained that is free from the shortcomings of classical measurement theories. The model uses the corrected model of S. Stevens and the reflection principle of J. Barzilai.

The measurement model was tested using laws that were obtained empirically. Using the model it is shown that Fechner's empirical law is equivalent to Stevens's empirical law. This means that the problem which has attracted attention of many researchers for almost a century, has been solved.

A numerical example demonstrates the possibilities of the proposed measurement model. It is shown that the model can be used for extended analysis of expert assessments.

Keywords: measurement theory, Fechner's law, Stevens' law, Rasch model, concept of meaningfulness.

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Модель измерения неаддитивной величины

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Предложена модель измерения неаддитивной величины, в частности модель субъективного измерения. Целью данной работы являлось развитие теории измерений и формирование модели субъективного измерения. Для обоснования модели использована скорректированная модель Стивенса.

Рассмотрена обобщенная структура модели измерения, которая включает эмпирическую систему, математическую систему и гомоморфизм эмпирической системы в числовую систему. Установлено, что основными недостатками классических теорий измерения являются: 1) гомоморфизм не отображает операции в системах, что позволило бы говорить об осмысленности теоретической модели измерений; 2) отсутствует модель эмпирического измерения, которая могла бы подтвердить существование гомоморфизма. Для преодоления недостатков существующих теорий определено уравнение измерения, связывающее результаты отображения эмпирической операции в числовую, а также сформулирована модель эмпирического измерения. Для построения модели измерения предложено использовать скорректированную модель Стивенса, которая дополнена принципом отражения Дж. Барзилая. В основу модели количественного измерения положены два способа измерений, с помощью которых эмпирически измеряется особый параметр – рейтинг, связанный с разностью или отношением искомых значений величины. Обосновано предположение о том, что оба способа измерения можно использовать совместно для измерения одной и той же величины. Причём результаты измерения будут в определённом смысле эквивалентны.

Показано, что такой подход позволяет получить модель количественного измерения, которая свободна от недостатков классических теорий измерения. Сформулирован алгоритм количественного измерения, а также принцип отражения, обеспечивающий соответствие эмпирической и числовой систем модели.

Предложенная модель измерения подтверждена эмпирически. С её помощью показано, что эмпирический закон Фехнера эквивалентен эмпирическому закону Стивенса. Тем самым получено решение классической проблемы субъективного измерения.

На конкретном примере продемонстрированы возможности предложенной модели измерения. Показано, что модель можно использовать для расширенного анализа экспертных оценок.

Ключевые слова: теория измерений, закон Фехнера, закон Стивенса, модель Раша, концепт осмысленности.

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Introduction

Measurement theory permits us to consider both objective and subjective measures from a unified point of view. Objective measures are associated with metrology [1]. Metrology is the science of measuring. The basis of metrology is units of measurement. Metrology also includes measuring instruments. The theory of objective measures is well developed. The theory of subjective measurements is based on the opinions and assessments of experts and requires further development [2].

Measurement is currently referred to as the process of obtaining an experimental value or values of a quantity that can reasonably be attributed to a quantity [3]. Every science experiment should follow the basic principles of proper investigation. An objective experiment is carried out using technical devices. Subjective experiments are based on expert opinions, feelings, and general impressions. And, if the justification of an objective experiment is technical devices, then further development of the measurement theory is required to verify the adequacy of the results of the subjective measurements [4].

Measurement theory permits us to consider both objective and subjective measures from a unified point of view. Objective measures are associated with metrology. Metrology is the science of measuring. The basis of metrology is units of measurement. Metrology also includes measuring instruments. The theory of objective measures is well developed. The theory of subjective measurements is based on the opinions and assessments of experts and requires further development [5].

The additive representation of the measurement process assumes that the addition operation has an empirical meaning. Representative measurement theory was created to overcome the limitations of additive measurement theory [6–8], (Figure 1). The representational measurement theory was originated by S.S. Stevens and other scientists. Representational measurement theory is based on the properties of binary relations and defines measurement as a mapping between two relational structures, an empirical one and a numerical one. For simplicity, since algebraic operations can be reduced to relations without loss of generality, representative theory does not include algebraic operations.

Empirical system	Mapping	Mathematical system
Objects $A_1, A_2, A_1, \ldots$ Relationships (A_i, A_j)	$u_i = u(A_i)$ depends on the type of measurement scale	Values of the magnitude $u_1, u_2, u_3, \ldots$

Figure 1 – Model by representative measurement theory

S. Stevens (1946) believed that numerical values should be assigned to objects according to certain rules. A measurement scale is a classification that describes the assignment rules.

New trends have appeared in the theory of measurements, which should be taken into account to substantiate a model of measurement. For example, a mathematical model of an empirical system was considered [9–10]. The model for measuring is proposed in the papers [11–12]. Let the general measurement model (Figure 2) include an empirical system, a mathematical system, and a mapping from an empirical system to a mathematical system:

1. Empirical system. Objects of measurement $A_1, A_2, A_3, \ldots$ and pairs of objects (A_i, A_j).

2. Mathematical system. u_i is a numerical value, and $(u_i - u_j)$ is the operation result.

3. Mapping. Each object A_i maps to a value u_i and each pair of objects (A_i, A_j) maps to the operation results $(u_i - u_j)$.

Mathematical model of an empirical system	Mapping	Mathematical system
Objects $A_1, A_2, A_3, \ldots$	$u_i = u(A_i)$	Values of the magnitude $u_1, u_2, u_3, \ldots$
Ordered pairs (A_i, A_j)	Measurement result $R_{ij} = R(A_i, A_j)$	The result of the operation $R(A_i, A_j) = u_i - u_j$

Figure 2 – General measurement model

Objects are mapped to values by the function $u_i = u(A_i)$, and pairs of objects (A_i, A_j) are mapped to the difference of values $(u_i - u_j)$. Hence, there are two mappings (see Figure 2). Let the empirical system be an affine line. Let A_1, A_2 and A_3 are arbitrary points on a straight line, and (A_1, A_2), (A_2, A_3) and (A_1, A_3) are rigid rods (Figure 3). Let's measure the length of these rods (Figure 3).

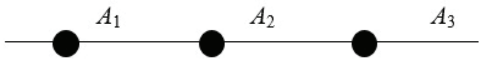

Figure 3 – Empirical system. The rod (A_1, A_3) consists of two rods (A_1, A_2) and (A_2, A_3)

The model for measuring the length of the rod (Figure 4) follows from the general measurement model (Figure 2).

Empirical system	Mapping	Mathematical system
Points on a straight line $A_1, A_2, A_3, ...$	$u_i = u(A_i)$	u_i – point coordinate values
Vectors $(A_1, A_2), (A_2, A_3), (A_1, A_3)$	Vector mapping $R_{ij} = R(A_i, A_j)$	Measurement equation $R(A_i, A_j) = u_i - u_j$

Figure 4 – Rod length measurement

Here the expression (A_i, A_j) means a vector. The point A_i is known as the start point, and the point A_j, is known as the end point. A vector is the result of an empirical measurement that characterizes the difference in position of two points on a straight line. Each vector (A_i, A_j) is assigned the value $R_{ij} = R(A_i, A_j)$. The formula $R(A_i, A_j) = u_i - u_j$ is used to calculate the measurement result. The formula $R(A_i, A_j) = u_i - u_j$ is used to determine the values of the quantity.

The measurement result of the vector (A_1, A_3) is equal to the sum of the measurement results of the vectors (A_1, A_2) and (A_2, A_3).

For the practical implementation of measurement, i. e., for empirical measurements, an appropriate model of measurement is needed. Stevens proposed a model in which he used a certain group of objects whose magnitude changed uniformly [4]. For example, in Figure 3, the position of points A_1, A_2 and A_3 on a straight line, changes uniformly. Then the vectors (A_1, A_2) and (A_2, A_3) are equal and, consequently, the measurement results of $R(A_1, A_2)$ and $R(A_2, A_3)$ coincide. Figure 5 shows the model of S.S. Stevens.

Empirical system	Mapping	Mathematical system
Vectors $(A_1, A_2), (A_2, A_3), (A_1, A_3)$	Vector mapping $R_{ij} = R(A_i, A_j)$	Measurement equation $R(A_i, A_j) = u_i - u_j$
Measurement $(A_1, A_2) = (A_2, A_3)$	Result mapping $R(A_1, A_2) = R(A_2, A_3)$	Measurement equation $u_1 - u_2 = u_1 - u_2$

Figure 5 – Empirical measurement model according to S. Stevens [4]

So far, the model for measuring the difference of values has been considered. A model for measuring the ratio of values can be obtained in a similar way. The Stevens model contains two measurement equations: for the difference and for the ratio of quantities. In the first case, the values are determined on a scale of intervals; in the second case, on a log-interval scale. S. Stevens used this model of measurement to classify measurement scales [4]. It only remains to add that the Stevens classification also needs to be corrected.

The aim of the work was to develop the theory of measurements based on the corrected model of measurements by S.S. Stevens. This work is a continuation of the work [11–12].

A critical analysis of the Stevens measurement model

The four scales were suggested by S.S. Stevens in 1946. Later, in 1957, S. Stevens added a fifth, the log-interval scale, but came to the conclusion that this scale was useless. And the logarithmic scale is no longer in use today. Stevens' model corresponds to the concept of realism. According to J. Michell [13–14], numbers are ratios between quantities and exist in space and time. An empirical relational system is posited as an objective, independently existing structure able to be numerically represented.

Such an empirical structure was considered in 1923 by the physicist A. Friedman. Following A. Friedman, let's axiomatically define "an exceptional group of objects that allows us to make a special evaluation". Let the objects $A_1, A_2, A_3, \ldots$ be sorted in ascending order of quantity, and the quantity of these objects changes uniformly; $u_i = u(A_i)$, where u_i is the value of the quantity; the differences in values $(u_{i+1} - u_i)$ are equal to each other: $u_2 - u_1 = u_3 - u_2 = \ldots = u_n - u_{n-1}$. In accordance with the definition of A. Friedman, such a special assessment is called a measurement. Difference values are defined using equality:

$$u_i - u_j = \lambda_1 (i - j), \tag{1}$$

where λ_1 is an unknown constant, $\lambda_1 > 0$. The values u_j are determined by a linear transformation, that is, on the interval scale.

Let $v_i = v(A_i)$, where v_i is the value of the quantity and the rations of the values are equal: $v_2/v_1 = v_3/v_2 = \ldots = v_n/v_{n-1}$. Then the ratios of values are determined by the formula:

$$\ln (v_i/v_j) = \lambda_2 (i - j), \tag{2}$$

where λ_2 is an unknown constant, $\lambda_2 > 0$. The logarithms of the values are determined up to a linear transformation, i. e., on the scale of log-intervals scale. As a result, two measurement equations are obtained (1) and (2), with two different measurement operations: subtraction and division. Values are determined on an interval scale and a log-interval scale. S.S. Stevens used a similar measurement model to classify measurement scales.

The concept of measurement scales looks convincing, and only the "unnecessary" fifth scale breaks the logic. S.S. Stephens thought a log scale was mathematically interesting, but it, like many mathematical models, has proven empirically useless. Such a claim is controversial. Let's take an example of measuring a non-additive quantity. Density is an example of a non-additive quantity. Let the density of the two samples be equal to 1 kg/m^3 and 2 kg/m^3. Then the sum of densities is not defined, but the ratio of densities is defined.

Example. Let the densities of samples A_1, A_2, A_3, A_4 and A_5 change uniformly. Density values can be measured in two ways. 1. The difference between two density values is calculated by the formula (1) $u_i - u_j = i - j$, where u_i are the values that characterize the density; $i, j = 1, 2, ..., 5$; $\lambda_1 = 1$. The ratios of density values satisfy the equality $v_{i+1} / v_i = 2$, where v_i are the density values. To calculate the ratios, use the formula $(v_i / v_j) = (2^i / 2^j)$; $i, j = 1, 2, ..., 5$.

Density values u_i are determined up to a constant factor, while values v_i are determined up to an arbitrary constant. In a particular case, the values are given in Table 1. The values have a natural interpretation. For example, the third sample ($i = 3$) has a density four times greater than the first, or two orders of magnitude greater than the first.

Table 1

The density values are obtained on the interval and log-interval scales

Interval scale of "density" values u_i	1	2	3	4	5
Log-interval scale of density values v_i	2	2^2	2^3	2^4	2^5

The example confirms that if the value of objects A_1, A_2, ... changes uniformly, it is reasonable to consider two measurement scales: the intervals scale and the log-intervals scale (Table 1). Stevens believed that the scale of logarithmic intervals was useless [4]. But density is not defined on the scale of relations since density is a non-additive quantity. The density is determined on the logarithmic scale of intervals. Therefore, there is reason to believe that the Stevens model requires adjustment.

The measurement model (the adjusted Stevens model)

From equalities (1) and (2), it follows that the interval scale values and log interval scale values are interconnected by the formula:

$$(u_i - u_j) = \lambda \ln(v_i/v_j), \tag{3}$$

where $i, j = 1, 2, \ldots, n$; u_i and v_i are the values of the quantity; $\lambda = \lambda_2/\lambda_1$. It is straightforward to demonstrate that equality (3) is satisfied for the values u_i and v_i in Table 1.

Equality (3) means that the mapping $u = \ln(v)$ preserves the measurement operation: the ratio of values maps to the difference of values. In addition, for the values u_i and v_i, there is a one-to-one correspondence between the values of u_i and v_i using the mapping $u = \ln(v)$. The mapping $u = \ln(v)$ is an isomorphism of two algebraic structures: the set of positive integers under the operation of division, onto the set of real numbers under the operation of subtraction. As a result, isomorphic structures cannot be distinguished from one another solely on the basis of structure; they are equivalent [15].

During the measurement process each pair of objects is assigned a value $(u_i - u_j)$ or (v_i/v_j). This means that the result of an empirical measurement is equal to the result of an arithmetic operation and not the value of the quantity. To unify the measurement process, it is convenient to introduce a rating definition based on equality (3):

$$R_{ij} = \lambda_1(u_i - u_j); \quad (4)$$

$$R_{ij} = \lambda_2 \ln(u_i/v_j), \quad (5)$$

where $i, j = 1, 2, \ldots n$. The quantity values are u_i and v_i, $v_i > 0$, and the positive constants are λ_1, λ_2.

For objects whose quantity changes uniformly, the rating is determined up to a constant factor λ:

$$R_{ij} = \lambda(i - j). \quad (6)$$

Such a definition of the rating will be called classical. The classic definition of rating follows from the Stevens measurement model. The rating does not depend on the choice of measurement model (5) or (6). A direct check shows that the rating values satisfy the consistency condition:

$$R_{ij} = R_{ik} + R_{kj}. \quad (7)$$

The axiomatic model of measurement includes the compatibility condition (7) and two measurement models (4) and (5), where u_i and v_i are values, and R_{ij} are rating values. Let the values of the quantity be on the interval scale if they are the solution of the system of equations (5), and on the logarithmic scale if they are the solution of the system of equations (6). The ratio scale is an interval scale modified to include an inherent zero starting point. The ratio scale is an auxiliary scale.

As a result, a theoretical measurement model was obtained, which can be used for both subjective and objective measurements. For the measurement models the measurement algorithm is:

1. Select the measurement model (4) or (5).
2. Find the measurement results $(u_i - u_j)$ or (v_i/v_j).
3. Calculate the rating R_{ij}.
4. Check the compatibility conditions (7).
5. Select the measurement equation (4) or (5) and find the values of the measured quantity.

The values of the quantity are defined in the scale of intervals if they are the solution of the system of equations (5), and in the scale of log-intervals if they are the solution of the system of equations (5). The ratio scale is a scale of intervals in which the zero element, the reference point, is defined. The ratio scale is an auxiliary scale.

In addition, the measurement model follows the principles:

1. The principle of reflection. Operations within the mathematical system are applicable if and only if they reflect corresponding operations within the empirical system.
2. The principle of equivalence. The interval scale and the log-interval scale are equivalent.

From the equivalence principle, organically follows:

1. Fechner's law in the form of paired comparisons [11].
2. Stevens' law in the form of paired comparisons [11].
3. Rasch model [16].

Stevens' Experimental Law (1947) was proposed to replace Fechner's Experimental Law (1848). The contradiction between the laws of Fechner and Stevens still exists. The proposed model measurement solves this problem. In addition, the experimental laws of psychophysics follow from the measurement model (5) and (6). Thus, the measurement model has strong empirical support.

An example implementation of a quantification model

Five samples of the drinks are evaluated by seven experienced experts (ISO 11056). Drinks contain different amounts of caffeine. Let A_i be a coffee brand, k be the expert's serial number, and u_{ki} be assessments of the coffee brand. Table 2 shows the assessments of brands, u_{ki}.

Table 2

Data related to the five samples

v_{ki}	A_1	A_2	A_3	A_4	A_5
1	10	20	35	40	70
2	8	20	38	44	85
3	8	20	36	40	75
4	7	15	32	37	70
5	12	25	38	40	75
6	12	22	35	40	80
7	9	18	35	40	84

The values were assigned based on the relation; if an attribute is twice as intense, it has been assigned a value twice as high. The assessment can be considered as a measurement on the log interval scale. Individual r_k ratings for each expert are calculated using the formula $r_{ki} = \ln(v_{ki}/v_{k1})$.

Table 3

Individual rating values

v_{ki}	A_1	A_2	A_3	A_4	A_5
1	0.00	0.69	1.25	1.39	1.95
2	0.00	0.92	1.56	1.70	2.36
3	0.00	0.92	1.50	1.61	2.24
4	0.00	0.76	1.52	1.67	2.30
5	0.00	0.73	1.15	1.20	1.83
7	0.00	0.61	1.07	1.20	1.90

The group rating R (Table 4) is calculated as the average of individual ratings (see Table 3) for each brand of coffee. A criterion for the consistency of expert assessments is proposed: the significance of the correlation coefficients ρ, $\rho_k = \rho(r_k, R)$. Correlation coefficients according to Student's t-test are significant with a significance level of 0.05. Therefore, the hypothesis of the consistency of expert assessments is accepted.

Table 4

Group assessment of the rating

A	A_1	A_2	A_3	A_4	A_5
R	0.00	0.76	1.35	1.47	2.10

The rating measurements can be used in demand forecasting and sales planning models. Suppose that the expert consistently compares all brands with the first one. Let p_i be the probability of choosing brand A_i in this situation. Then the ratios of probabilities p_i/p_1 are related to the rating by formula (6), which we write as:

$$R_i = \lambda \ln(p_i/p_1), \tag{8}$$

where λ for formula (7) can be found by using additional information. Formula 7 is the Rasch model [16].

The example shows that the measurement results can be interpreted using the rating definition. In the example under consideration, the scale of log intervals was chosen based on the recommendations for conducting such studies. To confirm that the measurement scale is log-interval, it is necessary to check (at least partially) the compatibility condition (3).

Conclusion

The measurement of non-additive quantities is a problem that was considered in this article. For example, subjective measurements are measurements of non-additive quantities. The analysis of modern works on the theory of measurements shows that this problem is still relevant. These problems are considered in the works of J. Barzilai and J. Michel. It has been established that there is no measurement equation in measurement theory that defines the natural connection between the empirical and mathematical systems.

The concept of realism has been applied to the formation of measurement models. In particular, this means that empirical structures that support measurement must naturally produce real numbers. The realistic principle for obtaining scale values is formed on the basis of the Stevens model. The Stevens model is the rationale for the classification of measurement scales. However, the analysis of the Stevens model showed that it needs to be refined.

Taking into account the concept of realism, a model of quantitative measurement is proposed. This model was used by S.S. Stevens for the classification of measurement scales.

The model includes two measurement operations. The result of a measurement operation is a difference or ratio of values. The definition of the rating allows you to consider both measurement operations at the same time. The rating is a generalized result of the measurement, which does not depend on the

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choice of the measurement operation. The assumption that both measurement operations can be used together to measure the same quantity is substantiated. Moreover, the measurement results in this case are equivalent. On this basis, the principle of equivalence is formulated.

An algorithm for quantitative measurement is formulated, as well as a reflection principle that ensures the correspondence between the empirical and mathematical systems.

The proposed model of measurement has convincing experimental confirmation. The model eliminated the contradiction between the empirical laws of Fechner and Stevens. It is shown that they are equivalent.

The definition of the measurement equation is given. The measurement equation maps an empirical system into a mathematical system. From the measurement equation follows the definition of the measurement scale. In general, the concept of measurement has been formed, which considers subjective and objective measurements from a single point of view.

An example of the application of the measurement model is given. It is shown that an extended analysis of expert assessments can be performed using a measurement model. Such an analysis can be used to solve the problem of forecasting supply and demand in the economy.

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